MATH7021: Assignment 1 — Remarks for Poorly Answered Questions

March 6, 2024

1 General Remarks

- given that ye had the time, it would have been a good idea to check your answers by substitution. This would have saved a lot of us lost marks. It is very easy to solve *one* of the equations (with incorrect values) — so if you are checking, check that *all* equations are satisfied.
- Note if you are using rounding when doing Gaussian Elimination) you <u>have to do partial pivoting</u>.
- Let me be clear because some of this didn't get this:
  - Gaussian Elimination is very sensitive[1] to ROUNDING ERROR, therefore
  - when you are doing Gaussian Elimination you may NOT use rounding. However,
  - if you *must* use rounding (such as when you are on a computer; i.e. Excel), you MUST DO PARTIAL PIVOTING.

[1]what this means is that if you round you can get very different solutions

2 Problem B: Illegal 'Row-Operations'

Consider a 2×2 linear system given by

$$\begin{aligned} x + 2y &= c_1 \\ 3x + 4y &= c_1 + 10 \end{aligned}$$

Written in augmented matrix form this is given by:

$$\left[\begin{array}{cc|c} 1 & 2 & c_1 \\ 3 & 4 & c_1 + 10 \end{array}\right]. \tag{1}$$

1. Suppose we do the 'row operation' $r_2 \to r_2 \times r_1$:

$$\left[\begin{array}{cc|c} 1 & 2 & c_1 \\ 3 & 8 & c_1^2 + 10c_1 \end{array}\right].$$

This corresponds to the linear system:

$$\begin{aligned} x + 2y &= c_1 \\ 3x + 8y &= c_1^2 + 10c_1 \end{aligned} \tag{2}$$

(c) What can you conclude about 'row operations' of the form $r_i \to r_i \times r_j$?

Remark: Some of us didn't notice that the solution to linear system (2) was different to the solution to linear system (1). Therefore you can't multiply rows together when doing Gaussian Elimination. Similarly for Q. 2.

(b) Starting with the linear system (1), suppose we do the row operation $r_2 \to r_2 + 1$:

$$\left[\begin{array}{cc|c} 1 & 2 & c_1 \\ 4 & 5 & c_1 + 11 \end{array}\right].$$

This corresponds to the linear system:

$$\begin{aligned} x + 2y &= c_1 \\ 4x + 5y &= c_1 + 11 \end{aligned} \tag{3}$$

ii. What can you conclude about 'row operations' of the form $r_i \to r_i + k$?

Remark: Same as above, some of us didn't notice that the solution to linear system (3) was different to linear system (1). You cannot add a constant to a row.

3 Problem C: Gaussian Elimination: Abstract Problems

1. * Using three-decimal-place rounding, use *Microsoft Excel* to solve the linear system:

$$\begin{pmatrix} 1.80 & 2.52 & 4.50 \\ 1.60 & 5.12 & 5.44 \\ 2.50 & 6.50 & 9.25 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}$$

Remark: Gaussian Elimination without partial pivoting is very susceptible to rounding error. Therefore if you are rounding when doing Gaussian elimination you *must* use partial pivoting. You have to swap rows in order to avoid dividing by small numbers: so the first row operations should have been $r_1 \leftrightarrow r_3$.

3.1 Problem E: Truss Systems: the Method of Joints

In this question we investigate the *Method of Joints*. The theory says that if you compare the number of beams and reactions $(b+r)$ and twice the number of joints $(2j)$ you have (with some subtleties I won't go into:)

- $b+r=2j$ implies that the system is *determinate*
- $b+r>2j$ implies that the system is *indeterminate*
- $b+r<2j$ implies that the system is *unstable*

3. Consider the following truss:

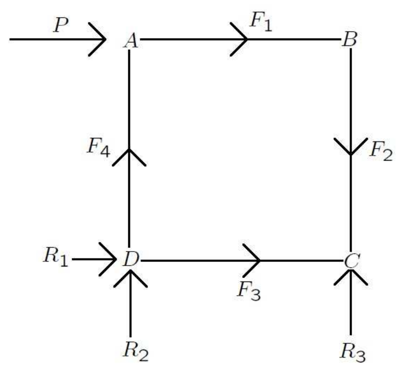

Figure 1: There is an external force of P on the truss as shown. The truss is fixed at D so there are two reactions there. There is a roller at C so only one reaction as shown.

In this case we have $b+r=4+3=7$ and $2j=8$ so that $b+r<2j$ suggesting that the system is unstable.

The equations governing the forces and reactions, using the method of joints, when put in augmented matrix form (first the F_i and then the R_i) are:

$$\left[\begin{array}{ccccccc|c}
1 & 0 & 0 & 0 & 0 & 0 & 0 & -P \\
0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right]$$

(c) <u>What assumption was made to allow us to write down the equations governing the truss system?</u>

Remark: I am not sure if everyone understood this 100%... to write the equations we assume that the joints are in equilibrium and so the truss is

$$\text{STABLE.}$$

If a joint is in equilibrium you have two equations:

$$\sum(\text{horizontal forces}) = 0$$
$$\sum(\text{vertical forces}) = 0$$

Note

- all determinate trusses are stable
- all indeterminate trusses are stable, so
- not all stable trusses are determinate, but note
- indeterminate does not mean unstable

(d) In this case we have

"assumption" implies "equations" implies "nonsense".

Therefore the "assumption" must have been incorrect. <u>Hence, comment on the stability of the truss.</u>

Remark: We assumed that the truss was stable to write down the equations. The equations implied an absurdity. Therefore the assumption that the truss was stable is incorrect and so the truss is unstable.

4. Complete the following. I use the word *suggests* because things aren't always as straightforward as the above three examples.

   Remark: I agree that the first part is hard on its own however if you go through the three points that suggests what the correct words for the first part were.

   > *A truss system composed of b BEAMS and r REACTIONS subject to an external FORCE has $b + r$ VARIABLES.*
   >
   > *Using the method of JOINTS, for each JOINT in a truss system we can write down two EQUATIONS. Therefore if there are j joints in the truss system the associated linear system has $2j$ EQUATIONS.*
   >
   > *If $b + r = 2j$, then the number of VARIABLES/COLUMNS equals the number of EQUATIONS/ROWS. This suggests that when the linear system is brought into row-reduced form that every COLUMN has a pivot. Thus we have a UNIQUE solution and so the truss system is determinate.*
   >
   > *If $b + r > 2j$ then the number of VARIABLES/COLUMNS is greater than the number of EQUATIONS/ROWS. This suggests that when the linear system is brought into row-reduced form we have PIVOTLESS columns. We know from class that pivotless columns correspond to PARAMETERS. Therefore, in this case we have at least one PARAMETER so there are an INFINITE number of solutions. In this case the truss system is said to be indeterminate.*
   >
   > *If $b + r < 2j$ then the number of VARIABLES/COLUMNS is less than the number of EQUATIONS/ROWS so that when the linear system is brought into row-reduced form there is a chance that there are NO solutions. In this case, the assumption that the truss system is STABLE leads to an absurdity so that we must conclude that the truss system is UNSTABLE.*