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CIT Autumn Examinations 2018/19

Note to Candidates: Check the Programme Title and the Module Description to ensure that you have received the correct examination. If in doubt please contact an Invigilator.

Module Title: **Discrete Maths**

Module Code: **MATH6004**

Programme Title(s):
BSc Software Development Y1
BSc Hons Computer Systems Y1
BSc (Hons) IT Management Y1
BSc Information Technology Y1
BSc (Hons) Software Devel Y1
BSc (Hons) Web Development Y1
HC Software Dev Y1 ACCS

Block Code(s):
KCOMP_7_Y1 **KDNET_8_Y1** **KITMN_8_Y1**
KITSP_7_Y1 **KSDEV_8_Y1** **KWEBD_8_Y1**
KCOME_6_Y1

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Internal Examiner(s): Dr. Marie Nicholson, Dr. Michael Brennan, Dr. Justin Mc Guinness, Mr. Adrian O Connor, Dr. Robert Heffernan

Instructions: Answer all four questions.

Duration: 2 hours

Required Items:

Question 1.

(a) The number of function calls $C(n)$ to a recursive function designed to calculate Fibonacci numbers is given by the following recursive definition.

$$C(n) = \begin{cases} 1 & \text{if } n = 0 \\ 1 & \text{if } n = 1 \\ C(n-1) + C(n-2) + 1 & \text{if } n > 1 \end{cases}$$

i. List the first 6 terms of this sequence.

ii. Write $C(100)$ in terms of $C(998)$ and $C(997)$.

[7 marks]

(b) Consider the sequence

$$12, 16, 20, 24, 28, 32, 36, \ldots$$

i. Give a recurrence relation that describes this sequence.

ii. Is this an arithmetic or geometric sequence?

iii. Sum the first 100 terms.

[10 marks]

(c) A charitable fund, set up to alleviate the suffering of unwanted pets at Christmas, gives an annual donation of €50,000 to pet shelters at the end of each year.

Let $F(n)$ be the amount in the fund at the end of the year after the donation has been paid.

The fund is invested at 3% p.a. and is valued at €2,000,000 at the end of year 0 after the donation has been paid.

i. Write down a recurrence relation for the value of the fund at the end of each year after the donation has been paid.

ii. The benefactor did some spreadsheet calculations and decided that the donation should increase annually by €100. Adjust your recurrence relation for the fund value to reflect the revised level of donation.

[8 marks]

Question 2.

(a) Write the following argument in symbols. State whether it represents a valid argument or a fallacy. Quote the appropriate rule of inference or fallacy.

If you gamble, then you're stupid. You don't gamble. Therefore you're not stupid.

[6 marks]

(b) Construct a truth table for the logical expression

$$(p \vee (q \wedge r)) \to \neg p.$$

[11 marks]

(c) Find truth values for p, q and r given that

$$[(p \to q) \wedge (q \wedge r)] \to (r \to p)$$

is false. **[8 marks]**

Question 3.

(a) Justify each of the following conclusions with either an equivalence rule or an inference rule.

i. *Ciara is both athletic and intelligent. Therefore, Claire is athletic.*

ii. *Liam has never been to Cork or Donegal. In other words, Liam has never been to Cork and Liam has never been to Donegal.*

[4 marks]

(b) Use the equivalence rules of logic to show that the following proposition is a tautology.

$$(q \wedge \neg p) \rightarrow q$$

[12 marks]

(c) Write a proof sequence for the following assertion. Justify each step.

$$\begin{array}{rl} & p \vee q \\ & \neg p \\ & q \rightarrow r \\ \hline \therefore & q \wedge r \end{array}$$

[9 marks]

Question 4.

(a) Let

$$A = \begin{bmatrix} 1 & 2 \end{bmatrix}, B = \begin{bmatrix} 3 \\ 4 \end{bmatrix}, C = \begin{bmatrix} 1 & 5 \\ 0 & -2 \end{bmatrix}$$

Calculate $BA - 2C$. **[8 marks]**

(b) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation that represents a reflection about the x-axis.

i. Write the standard matrix M for the transformation.

ii. Is the transformation invertible? Explain your answer.

iii. Find $T\begin{pmatrix} 1 \\ 2 \end{pmatrix}$, $T\begin{pmatrix} -4 \\ -4 \end{pmatrix}$ and $T\begin{pmatrix} x \\ y \end{pmatrix}$.

[8 marks]

(c) Consider the system

$$\begin{aligned} 2x + 4y &= 8 \\ x + 2y &= 4 \end{aligned}$$

i. Write this system as a matrix equation in the form $A\vec{x} = \vec{b}$.

ii. Find the determinant of A. Does A^{-1} exist?

iii. Does this system have a unique solution, no solution or infinitely many solutions? Justify your answer. Write the solution(s) if there are any.

[9 marks]

Logical equivalences

Conditional / Implication $A \to B \equiv \neg A \vee B$ and $A \to B \equiv \neg(A \wedge \neg B)$

Biconditional $A \leftrightarrow B \equiv (A \to B) \wedge (B \to A)$

De Morgan $\neg(A \wedge B) \equiv \neg A \vee \neg B$ and $\neg(A \vee B) \equiv \neg A \wedge \neg B$

Negation / Inverse $A \vee \neg A \equiv T$ and $A \wedge \neg A \equiv F$

Identity $A \wedge T \equiv A$ and $A \vee F \equiv A$

Double negation $\neg(\neg A) \equiv A$

Idempotent $A \wedge A \equiv A$ and $A \vee A \equiv A$

Commutative $A \wedge B \equiv B \wedge A$ and $A \vee B \equiv B \vee A$

Associative $(A \wedge B) \wedge C \equiv A \wedge (B \wedge C)$ and $(A \vee B) \vee C \equiv A \vee (B \vee C)$

Distributive $A \wedge (B \vee C) \equiv (A \wedge B) \vee (A \wedge C)$ and $A \vee (B \wedge C) \equiv (A \vee B) \wedge (A \vee C)$

Annihilation $A \vee T \equiv T$ and $A \wedge F \equiv F$

Absorption $A \vee (A \wedge B) \equiv A$ and $A \wedge (A \vee B) \equiv A$

Rules of inference

Name	**Tautology**	**Rule**
Modus ponens / Direct reasoning	$[A \wedge (A \to B)] \to B$	A $A \to B$ $\therefore\ B$
Modus tollens / Indirect reasoning	$[\neg B \wedge (A \to B)] \to \neg A$	$\neg B$ $A \to B$ $\therefore\ \neg A$
Hypothetical syllogism	$[(A \to B) \wedge (B \to C)] \to (A \to C)$	$A \to B$ $B \to C$ $\therefore\ A \to C$
Disjunctive syllogism	$[(A \vee B) \wedge \neg A] \to B$	$A \vee B$ $\neg A$ $\therefore\ B$
Addition	$A \to (A \vee B)$	A $\therefore\ A \vee B$
Simplification	$(A \wedge B) \to A$	$A \wedge B$ $\therefore\ A$
Resolution	$[(A \vee B) \wedge (\neg A \vee C)] \to (B \vee C)$	$A \vee B$ $\neg A \vee C$ $\therefore\ B \vee C$
Conjunction	$[(A) \wedge (B)] \to (A \wedge B)$	A B $\therefore\ A \wedge B$

Fallacies

Affirming the conclusion is incorrectly assuming that $[(A \to B) \wedge B] \to A$ is a tautology.

Denying the hypothesis is incorrectly assuming that $[(A \to B) \wedge \neg A] \to \neg B$ is a tautology.