

## Logical equivalences

**Conditional / Implication**  $A \rightarrow B \equiv \neg A \vee B$  and  $A \rightarrow B \equiv \neg(A \wedge \neg B)$

**Biconditional**  $A \leftrightarrow B \equiv (A \rightarrow B) \wedge (B \rightarrow A)$

**De Morgan**  $\neg(A \wedge B) \equiv \neg A \vee \neg B$  and  $\neg(A \vee B) \equiv \neg A \wedge \neg B$

**Negation / Inverse**  $A \vee \neg A \equiv T$  and  $A \wedge \neg A \equiv F$

**Identity**  $A \wedge T \equiv A$  and  $A \vee F \equiv A$

**Double negation**  $\neg(\neg A) \equiv A$

**Idempotent**  $A \wedge A \equiv A$  and  $A \vee A \equiv A$

**Commutative**  $A \wedge B \equiv B \wedge A$  and  $A \vee B \equiv B \vee A$

**Associative**  $(A \wedge B) \wedge C \equiv A \wedge (B \wedge C)$  and  $(A \vee B) \vee C \equiv A \vee (B \vee C)$

**Distributive**  $A \wedge (B \vee C) \equiv (A \wedge B) \vee (A \wedge C)$  and  $A \vee (B \wedge C) \equiv (A \vee B) \wedge (A \vee C)$

**Annihilation**  $A \vee T \equiv T$  and  $A \wedge F \equiv F$

**Absorption**  $A \vee (A \wedge B) \equiv A$  and  $A \wedge (A \vee B) \equiv A$

## Rules of inference

Name	Tautology	Rule
Modus ponens / Direct reasoning	$[A \wedge (A \rightarrow B)] \rightarrow B$	$\frac{A \quad A \rightarrow B}{\therefore B}$
Modus tollens / Indirect reasoning	$[\neg B \wedge (A \rightarrow B)] \rightarrow \neg A$	$\frac{\neg B \quad A \rightarrow B}{\therefore \neg A}$
Hypothetical syllogism	$[(A \rightarrow B) \wedge (B \rightarrow C)] \rightarrow (A \rightarrow C)$	$\frac{A \rightarrow B \quad B \rightarrow C}{\therefore A \rightarrow C}$
Disjunctive syllogism	$[(A \vee B) \wedge \neg A] \rightarrow B$	$\frac{A \vee B \quad \neg A}{\therefore B}$
Addition	$A \rightarrow (A \vee B)$	$\frac{A}{\therefore A \vee B}$
Simplification	$(A \wedge B) \rightarrow A$	$\frac{A \wedge B}{\therefore A}$
Resolution	$[(A \vee B) \wedge (\neg A \vee C)] \rightarrow (B \vee C)$	$\frac{A \vee B \quad \neg A \vee C}{\therefore B \vee C}$
Conjunction	$[(A) \wedge (B)] \rightarrow (A \wedge B)$	$\frac{A \quad B}{\therefore A \wedge B}$

## Fallacies

**Affirming the conclusion** is incorrectly assuming that  $[(A \rightarrow B) \wedge B] \rightarrow A$  is a tautology.

**Denying the hypothesis** is incorrectly assuming that  $[(A \rightarrow B) \wedge \neg A] \rightarrow \neg B$  is a tautology.