MATH7021: Assignment 1 — Remarks for Poorly Answered Questions

April 7, 2022

1 General Remarks

- given that ye had the time, it would have been a good idea to check your answers by substitution. This would have saved a lot of us lost marks. It is very easy to solve *one* of the equations (with incorrect values) — so if you are checking, check that *all* equations are satisfied.
- some of us are doing only a single row operation per 'frame'. This will be too slow for the exam. Look at the notes to see how many row operations it is reasonable to do 'at once'.
- A number of us applied the Gaussian Elimination algorithm and just stopped... no you have to finish off the problem then!
- Note that if you are doing Gaussian Elimination by hand you should use exact fractions and square roots rather a decimal approximation. If you are using rounding when doing Gaussian Elimination you have to do partial pivoting.
- Let me be clear because some of this didn't get this:
  - Gaussian Elimination is very senstive[1] to ROUNDING ERROR, therefore
  - when you are doing Gaussian Elimination you may NOT use rounding. However,
  - if you *must* use rounding (such as when you are on a computer; i.e. Excel), you MUST DO PARTIAL PIVOTING.

[1]what this means is that if you round you can get very different solutions

2 Problem A: 2×2 Linear Systems

Some of us didn't quite nail this... the following uses c_i rather than k_i:

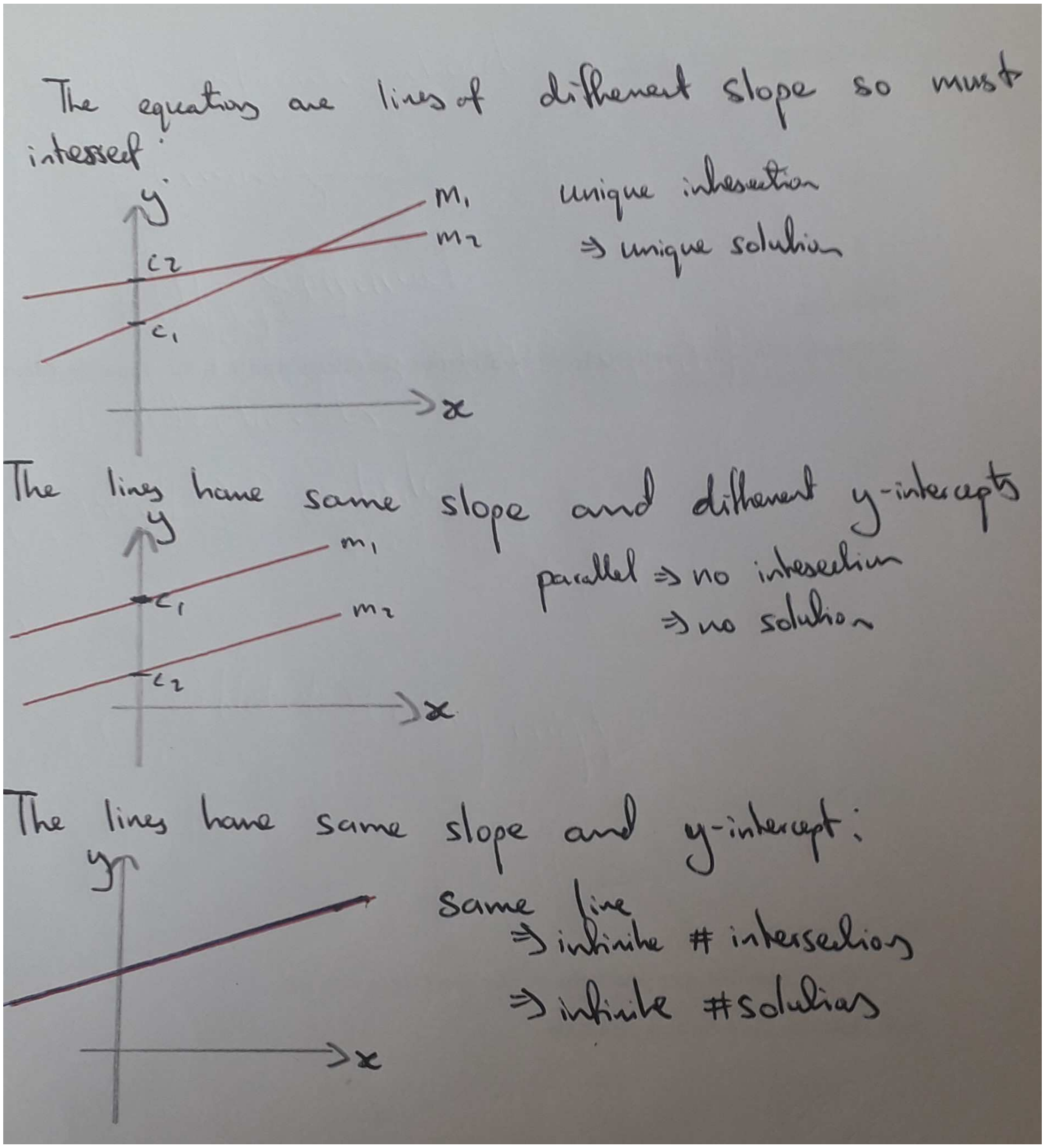

Figure 1: This is probably the best (and easiest) way to answer this question.

Remark: A number of us made a serious logical error here... take part (a). We need to prove the result for *ALL* systems such that $m_1 \neq m_2$. Showing the result for a single example, say $m_1 = 3$ and $m_2 = 5$, isn't sufficient to prove the result for all systems such that $m_1 \neq m_2$.

For example, suppose you were asked to prove the following 'theorem':

> *All Brazilians are good at soccer.*

Pointing out one Brazilian is good is not sufficient to prove that *all* Brazilians are good.

3 Problem C: Gaussian Elimination: Abstract Problems

1. * Using three-decimal-place rounding, use *Microsoft Excel* to solve the linear system:

$$\begin{pmatrix} 1.80 & 2.52 & 4.50 \\ 1.60 & 5.12 & 5.44 \\ 2.50 & 6.50 & 9.25 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}$$

Remark: Gaussian Elimination without partial pivoting is very susceptible to rounding error. Therefore if you are rounding when doing Gaussian elimination you *must* use partial pivoting.

Also it isn't sufficient just to 'pivot' the first column alone — if the second column needs it you must pivot a second time. For example, after pivoting once, making the one and then the zeroes underneath, most of us got to here:

$$\left[\begin{array}{ccc|c} 1 & 2.6 & 3.7 & 0.4c_3 \\ 0 & 0.96 & -0.48 & c_2 - 0.64c_3 \\ 0 & -2.16 & -2.16 & c_1 - 1.8c_3 \end{array}\right]$$

As -2.16 is bigger than 0.96 in magnitude the next row operation should have been $r_2 \leftrightarrow r_3$ not $r_2 \to r_2/0.96$ as most of us did.

3.1 Problem E: Truss Systems: the Method of Joints

In this question we investigate the *Method of Joints*. The theory says that if you compare the number of beams and reactions $(b + r)$ and twice the number of joints $(2j)$ you have (with some subtleties I won't go into:)

- $b + r = 2j$ implies that the system is *determinate*
- $b + r > 2j$ implies that the system is *indeterminate*
- $b + r < 2j$ implies that the system is *unstable*

3. Consider the following truss:

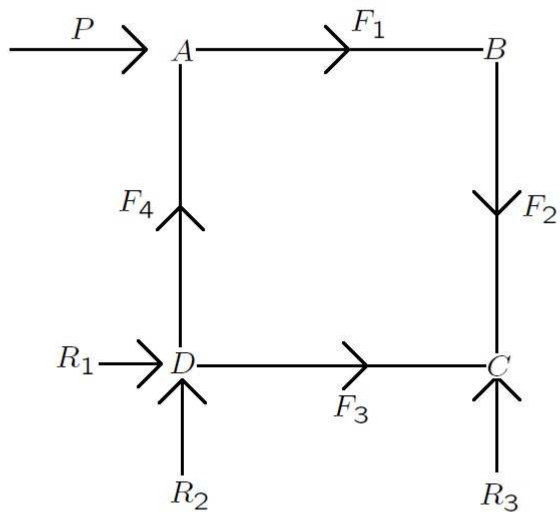

Figure 2: There is an external force of P on the truss as shown. The truss is fixed at D so there are two reactions there. There is a roller at C so only one reaction as shown.

In this case we have $b + r = 4 + 3 = 7$ and $2j = 8$ so that $b + r < 2j$ suggesting that the system is unstable.

The equations governing the forces and reactions, using the method of joints, when put in augmented matrix form (first the F_i and then the R_i) are:

$$\left[\begin{array}{ccccccc|c} 1 & 0 & 0 & 0 & 0 & 0 & 0 & -P \\ 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

(c) What assumption was made to allow us to write down the equations governing the truss system?

Remark: I am not sure if everyone understood this 100%... to write the equations we assume that the joints are in equilibrium and so the truss is

STABLE.

If a joint is in equilibrium you have two equations:

$$\sum(\text{horizontal forces}) = 0$$
$$\sum(\text{vertical forces}) = 0$$

Note

- all determinate trusses are stable
- all indeterminate trusses are stable, so
- not all stable trusses are determinate, but note
- indeterminate does not mean unstable

(d) In this case we have

"assumption" implies "equations" implies "nonsense".

Therefore the "assumption" must have been incorrect. Hence, comment on the stability of the truss.

Remark: We assumed that the truss was stable to write down the equations. The equations implied an absurdity. Therefore the assumption that the truss was stable is incorrect and so the truss is unstable.

4. Complete the following. I use the word *suggests* because things aren't always as straightforward as the above three examples.

   Remark: I agree that the first part is hard on its own however if you go through the three points that suggests what the correct words for the first part were.

   > *A truss system composed of b BEAMS and r REACTIONS subject to an external FORCE has $b + r$ VARIABLES.*
   >
   > *Using the method of JOINTS, for each JOINT in a truss system we can write down two EQUATIONS. Therefore if there are j joints in the truss system the associated linear system has $2j$ EQUATIONS.*
   >
   > *If $b + r = 2j$, then the number of VARIABLES/COLUMNS equals the number of EQUATIONS/ROWS. This suggests that when the linear system is brought into row-reduced form that every COLUMN has a pivot. Thus we have a UNIQUE solution and so the truss system is determinate.*
   >
   > *If $b + r > 2j$ then the number of VARIABLES/COLUMNS is greater than the number of EQUATIONS/ROWS. This suggests that when the linear system is brought into row-reduced form we have PIVOTLESS columns. We know from class that pivotless columns correspond to PARAMETERS. Therefore, in this case we have at least one PARAMETER so there are an INFINITE number of solutions. In this case the truss system is said to be indeterminate.*
   >
   > *If $b + r < 2j$ then the number of VARIABLES/COLUMNS is less than the number of EQUATIONS/ROWS so that when the linear system is brought into row-reduced form there is a chance that there are NO solutions. In this case, the assumption that the truss system is STABLE leads to an absurdity so that we must conclude that the truss system is UNSTABLE.*