MATH7021: Assignment 2 — Remarks for Poorly Answered Questions

April 27, 2022

1 General Remarks

- regardless of whether we are solving an ordinary differential equation (ode) where the independent variable is t (time) or x (distance), Laplace methods only apply to positive values of the independent variable; i.e. $t \geq 0$ or $x \geq 0$. Therefore if you are ever plotting a solution to an ode solved using Laplace, $t < 0$ or $x < 0$ should not be included (even if the formal 'solution' (e.g. $x(t) = \sin t$) makes sense for $t < 0$).
- given that ye had the time, it would have been a good idea to check your answers. This would have saved a lot of us lost marks. You can check your answers by seeing if

  A. they satisfy the differential equation[1],

  B. they satisfy the initial conditions.

  You will probably not have the time to check answers in the final exam.

Question-Specific Remarks

1. (a) ii. Show that we have $|x(t)| \leq 1$ for all t.

   Remark: We need to prove the result for *ALL* t. Showing the result for a few cases, say $t = 0, 1, 2$, isn't sufficient to prove the result for all possible times, t.

   For example, suppose you were asked to prove the following 'theorem':

   All Brazilians are good at soccer.

   Pointing out a few Brazilians that are good is not sufficient to prove that *all* Brazilians are good.

   The correct answer here is that

   $$x(t) = \cos(\omega t),$$

   for some ω; and for all $x \in \mathbb{R}$

   $$-1 \leq \cos x \leq 1;$$

   in other words cosine only takes values between -1 and 1. Therefore the absolute value of cosine is always between 0 and 1 and so $|x(t)| \leq 1$ for *all* t.

[1] Cf. Q.1(c) (ii)

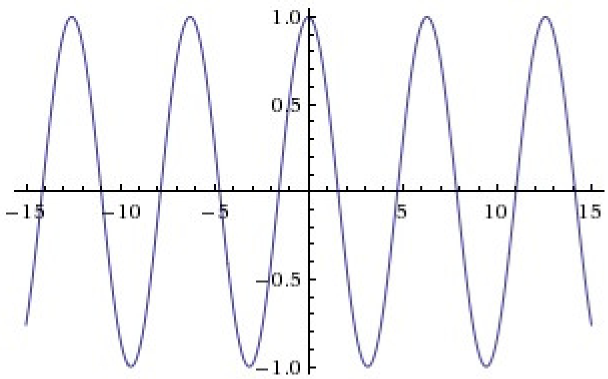

Figure 1: Cosine only takes on values between −1 and 1

iii. Explain this result $|x(t)| \leq 1$.

What I needed was something like:

The mass begins from rest at a distance of one from equilibrium. $|x(t)| \leq 1$ *means that the mass is never more than a distance of one from equilibrium and in fact oscillates between* $x = -1$ *and* $x = 1$.

Note with no damping there is no energy loss and the oscillations continue forever.

2. (a) Watch the video and write down:

   i. for what range of distances do we have underdamping
   ii. for what distance do we have critical damping
   iii. for what range of distances do we have overdamping?

   Those of us who answered:

   i. 20 to 2.5 mm
   ii. 2.5 to 1.5 mm
   iii. 1.5 to 0 mm

   or even

   i. 20 to 2.5 mm
   ii. 2 mm
   iii. 1.5 to 0 mm,

   did not get the full mark. You need to understand that CRITICAL damping only occurs at one distance and everything else is either overdamping or underdamping (no matter how insignificant); i.e. at 2.25 mm, although not shown in the video, is underdamped.

   (c) i. The biggest issue here was student's getting to, say,

$$Y(s) = \frac{\frac{4}{s} + 2s + 2}{(s+4)^2}.$$

   People from here went on to try and find the Rule II partial fraction expansion:

$$Y(s) \stackrel{!}{=} \frac{A}{s+4} + \frac{B}{(s+4)^2}.$$

You see on p.104 that to find a partial fraction expansion, that $Y(s)$ must be a *rational* function. This means that

$$Y(s)=\frac{p(s)}{q(s)},$$

with p and q *polynomials*, sums of positive powers of s. The top of $Y(s)$, $\frac{4}{s}+2s+2$ is *not* a polynomial because it is equal to

$$4s^{-1}+2s+2,$$

and the s^{-1} is not a positive power of s. There are various ways to avoid this, the easiest being to take the $Y(s)$ above and to multiply above and below by s:

$$Y(s)=\frac{\frac{4}{s}+2s+2}{(s+4)^2}\cdot\frac{s}{s}=\frac{4+2s^2+2}{s\cdot(s+4)^2},$$

showing that Y actually requires a Rule I and a Rule II.

From here the biggest issue was dealing with say,

$$\frac{p(s)}{s(s+4)^2}.$$

People wrote

$$\frac{p(s)}{s(s+4)^2}\overset{!}{=}\frac{A}{s}+\frac{B}{(s+4)^2},$$

and other wrong variants. A Rule I and a Rule II looks like:

$$\frac{p(s)}{s(s+4)^2}\overset{!}{=}\frac{A}{s}+\frac{B}{s+4}+\frac{C}{(s+4)^2}.$$

iii. <u>What type of damping will the bridge undergo if the wind suddenly stops?</u> If the wind stops then you are left with:

$$\frac{d^2y}{dt^2}+b\cdot\frac{dy}{dt}+c\cdot y(t)=0,$$

i.e. a damped harmonic oscillator. If we do the b^2-4ac analysis:

- $b^2-4ac<0\Rightarrow$ underdamped
- $b^2-4ac=0\Rightarrow$ critically damped
- $b^2-4ac>0\Rightarrow$ overdamped,

we can see the damping is critical.

You can also see this by looking at the solution:

$$y(t)=Be^{-at}+Cte^{-at}+A.$$

There are no oscillations — with no sine nor cosine there are no oscillations — and we don't have a two speed convergence (two exponentials). Therefore it is critically damped.