

MATH7021: Assignment 1 — Remarks for Poorly Answered Questions

April 4, 2019

1 General Remarks

- given that ye had the time, it would have been a good idea to check your answers by substitution. This would have saved a lot of us lost marks. It is very easy to solve *one* of the equations (with incorrect values) — so if you are checking, check that *all* equations are satisfied.
- Note that if you are doing Gaussian Elimination by hand you should use exact fractions and square roots rather a decimal approximation. If you are using rounding when doing Gaussian Elimination you have to do partial pivoting.
- Let me be clear because some of this didn't get this:
 - Gaussian Elimination is very sensitive¹ to ROUNDING ERROR, therefore
 - when you are doing Gaussian Elimination you may NOT use rounding. However,
 - if you *must* use rounding (such as when you are on a computer; i.e. Excel), you MUST DO PARTIAL PIVOTING.

2 2×2 Linear Systems

Remark: A number of us made a serious logical error here... take part (a). We need to prove the result for *ALL* systems such that $m_1 \neq m_2$. Showing the result for a single example, say $m_1 = 3$ and $m_2 = 5$, isn't sufficient to prove the result for all systems such that $m_1 \neq m_2$.

For example, suppose you were asked to prove the following 'theorem':

All Brazilians are good at soccer.

Pointing out one Brazilian is good is not sufficient to prove that *all* Brazilians are good.

¹what this means is that if you round you can get very different solutions

2.1 Illegal ‘Row-Operations’

Consider a 2×2 linear system given by

$$\begin{aligned}x + 2y &= c_1 \\ 3x + 4y &= c_1 + 10\end{aligned}$$

Written in augmented matrix form this is given by:

$$\left[\begin{array}{cc|c} 1 & 2 & c_1 \\ 3 & 4 & c_1 + 10 \end{array} \right]. \quad (1)$$

(a) Suppose we do the ‘row operation’ $r_2 \rightarrow r_2 \times r_1$:

$$\left[\begin{array}{cc|c} 1 & 2 & c_1 \\ 3 & 8 & c_1^2 + 10c_1 \end{array} \right].$$

This corresponds to the linear system:

$$\begin{aligned}x + 2y &= c_1 \\ 3x + 8y &= c_1^2 + 10c_1\end{aligned} \quad (2)$$

iii. What can you conclude about ‘row operations’ of the form $r_i \rightarrow r_i \times r_j$?

Remark: Some of us didn’t notice that the solution to linear system (2) was different to the solution to linear system (1). Therefore you can’t multiply rows together when doing Gaussian Elimination. Similarly for part (b).

(b) Starting with the linear system (1), suppose we do the row operation $r_2 \rightarrow r_2 + 1$:

$$\left[\begin{array}{cc|c} 1 & 2 & c_1 \\ 4 & 5 & c_1 + 11 \end{array} \right].$$

This corresponds to the linear system:

$$\begin{aligned}x + 2y &= c_1 \\ 4x + 5y &= c_1 + 11\end{aligned} \quad (3)$$

ii. What can you conclude about ‘row operations’ of the form $r_i \rightarrow r_i + k$?

Remark: Same as above, some of us didn’t notice that the solution to linear system (3) was different to linear system (1). You cannot add a constant to a row.

3 Gaussian Elimination: Abstract Problems

(a) * Using three-decimal-place rounding, use *Microsoft Excel* to solve the linear system:

$$\begin{pmatrix} 1.80 & 2.52 & 4.50 \\ 1.60 & 5.12 & 5.44 \\ 2.50 & 6.50 & 9.25 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}$$

Remark: Gaussian Elimination without partial pivoting is very susceptible to rounding error. Therefore if you are rounding when doing Gaussian elimination you *must* use partial pivoting.

Also it isn't sufficient just to 'pivot' the first column alone — if the second column needs it you must pivot a second time. For example, after pivoting once, making the one and then the zeroes underneath, most of us got to here:

$$\left[\begin{array}{ccc|c} 1 & 2.6 & 3.7 & 0.4c_3 \\ 0 & 0.96 & -0.48 & c_2 - 0.64c_3 \\ 0 & -2.16 & -2.16 & c_1 - 1.8c_3 \end{array} \right]$$

As -2.16 is bigger than 0.96 in magnitude the next row operation should have been $r_2 \leftrightarrow r_3$ not $r_2 \rightarrow r_2/0.96$ as most of us did.

3.1 Truss Systems: the Method of Joints

- (d) Complete the following. I use the word *suggests* because things aren't always as straightforward as the above three examples.

Remark: I agree that the first part is hard on its own however if you go through the three points that suggests what the correct words for the first part were.

- A truss system composed of b BEAMS and r REACTIONS subject to an external FORCE has $b + r$ VARIABLES.

Using the method of JOINTS, for each JOINT in a truss system we can write down two EQUATIONS. Therefore if there are j joints in the truss system the associated linear system has $2j$ EQUATIONS.

- If $b + r = 2j$, then the number of VARIABLES equals the number of EQUATIONS. This suggests that when the linear system is brought into row-reduced form, there are no COLUMNS without PIVOTS. Therefore there are no parameters, and hence a UNIQUE solution. Therefore the truss system is determinate.
- If $b + r > 2j$ then the number of VARIABLES is greater than the number of EQUATIONS. This suggests that when the linear system is brought into row-reduced form we have that the number of COLUMNS/VARIABLES is greater than the number of non-zero rows. We know from class that the number of PARAMETERS is equal to the number of COLUMNS/VARIABLES less the number of NON-ZERO/PIVOT rows so in this case we have at least one PARAMETER so there are an INFINITE number of solutions. In this case the truss system is said to be indeterminate.
- If $b + r < 2j$ then the number of VARIABLES is less than the number of EQUATIONS so that when the linear system is brought into row-reduced form there is a chance that there are NO solutions. In this case the assumption that the truss system is STABLE leads to an absurdity so that we must conclude that the truss system is UNSTABLE.