CORK INSTITUTE OF TECHNOLOGY

INSTITIÚID TEICNEOLAÍOCHTA CHORCAÍ

Sample Exam Examinations 2018/2019

Module Title: Sample Exam

Module Code:	MATH6055
School:	Computer Science
Programme Title:	Computer Science – Year 1
Programme Code:	
External Examiners(s):	The extern examiner
Internal Examiners(s):	Your lecturers
Instructions:	Answer **all** questions. Do not write, draw or underline in RED. Show all calculations and workings in full.
Duration:	2 Hours
Sitting:	Sample Exam
Requirements for this examination:	Mathematical Tables

Note to Candidates:

Please check the Programme Title and the Module Title to ensure that you have received the correct examination paper. If in doubt please contact an Invigilator.

Question 1.

(a) (i) Expand so that there are no parentheses and then simplify as much as possible

$$(5x-4)^2$$

(4 marks)

(ii) Solve for x in the following equation

$$\frac{3}{x-2}+5=\frac{4}{x-2}$$

(5 marks)

(iii) Write $\frac{a^3}{a\sqrt{a}}$ in the form a^p , where p is a rational number. (4 marks)

(b) Solve for x in each of the following equations:

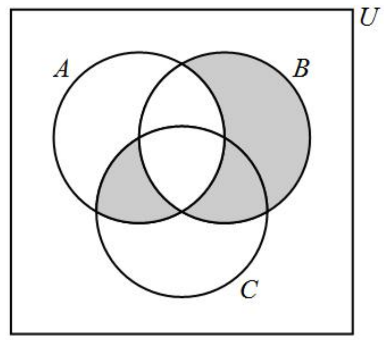
(i) $\log_{10}(x^2)=2.5$ (4 marks)

(ii) $5^{x+2}=12^{3-x}$ (4 marks)

(c) How many times you can divide an array of length n in half before you get down to single-element array? (4 marks)

Question 2.

(a) Use symbols to describe the shaded area in the following Venn diagram. (3 marks)



(b) Simplify $A \cup \overline{(B \cap \overline{A})}$ using the laws of sets. Indicate explicitly which laws are used at each step. (6 marks)

(c) Let C be the set $\{a,b\}$. Write out the elements of $C \times C$. How many elements are in $C \times C \times C$? (3 marks)

(d) Define a relation R on $\mathscr{P}(\{a,b,c,d\})$ by

$$(A,B) \in R \text{ if and only if } A \cap B = \emptyset$$

for $A,B \in \mathscr{P}(\{a,b,c,d\})$. Determine whether this relation is reflexive, symmetric, and/or transitive. Justify your answers. (6 marks)

(e) The relation R on $\{0,1,2,3\}$ given by

$$R = (0,0),(1,1),(2,2),(3,3),(0,1),(1,0),(2,3),(3,2)$$

is an equivalence relation (You do not need to show this).

(i) Draw a graph that represents R.

(ii) List the equivalence classes.

(7 marks)

Question 3.

(a) Let G be the graph with vertices $V = \{a, b, c, d\}$ and edges $E = \{(a, b), (a, c), (b, c), (c, d)\}$. Draw this graph. (5 marks)

(b) State a condition that guarantees that a graph has an Euler trail. If this condition is *not met* must it be the case that the graph does *not have* such a trail? (6 marks)

(c) For the graph below, answer the following questions.

(i) Give the degree of each vertex.

(ii) Is the graph connected? Give a reason for your answer.

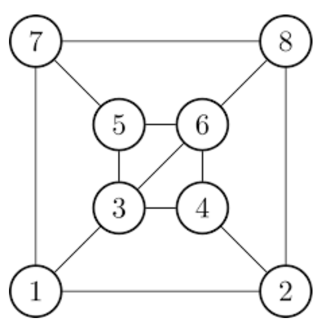
(iii) Is the graph a tree? Give a reason for your answer.

(iv) Does the graph have an Euler trail? If so, find one. If not, give a reason.

(v) Does the graph have an Euler circuit? If so, find one. If not, give a reason.

(vi) Does the graph have a Hamiltonian path? If so, find one.

(vii) Does the graph have a Hamiltonian cycle? If so, find one.



(14 marks)

Question 4.

(a) Let $X = \{1, 2, 3, \ldots, 10\}$. Define the successor "function" $S : X \to X$ by

$$S(n) = n + 1.$$

(i) Why is $S : X \to X$ not a function?

(ii) Give an example of a domain D such that $S : D \to D$ is a function.

(5 marks)

(b) Show that the function $f : \mathbf{N} \to \mathbf{N}$ given by $f(n) = n + 2$ in not onto. (5 marks)

(c) Let $X = \{1, 2, 3, 4\}$ and $Y = \{5, 6, 7, 8, 9\}$. Let $f = \{(1, 5), (2, 7), (4, 9), (3, 7)\}$.

(i) Is f a function?

(ii) Is f invertible? Explain your answer.

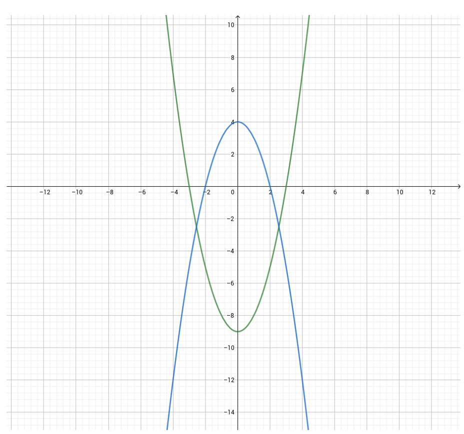
(5 marks)

(d) Let $X = \{1, 2, 3, 4\}$. Let $f : X \to \mathbb{R}$ be a function defined as the set of ordered pairs

$$\{(1, 2), (2, 3), (3, 4), (4, 5)\}.$$

Let $g : \mathbb{R} \to \mathbb{R}$ be the function defined as $g(x) = x^2$. List the ordered pairs of $g \circ f$. (5 marks)

(e) Below see a plot of the graphs of the $\mathbb{R} \to \mathbb{R}$ functions $f(x) = x^2 - 9$ and $g(x) = 4 - x^2$. Label appropriately:



(5 marks)

Indices and Logarithms

$a^p a^q = a^{p+q}$

$\dfrac{a^p}{a^q} = a^{p-q}$

$(a^p)^q = a^{pq}$

$a^0 = 1$

$a^{-p} = \dfrac{1}{a^p}$

$a^{\frac{1}{q}} = \sqrt[q]{a}$

$a^{\frac{p}{q}} = \sqrt[q]{(a)^p} = (\sqrt[q]{a})^p$

$(ab)^p = a^p b^p$

$\left(\dfrac{a}{b}\right)^p = \dfrac{a^p}{b^p}$

$\log_a(xy) = \log_a x + \log_a y$

$\log_a\left(\dfrac{x}{y}\right) = \log_a x - \log_a y$

$\log_a(x^q) = q\log_a x$

$\log_a 1 = 0$

$\log_a\left(\dfrac{1}{x}\right) = -\log_a x$

$a^x = y \Leftrightarrow \log_a y = x$

$\log_a(a^x) = x$

$a^{\log_a x} = x$

$\log_b x = \dfrac{\log_a x}{\log_a b}$

Sets

Name	Equality	
Double Complement Law	$\overline{(\overline{A})} = A$	
Identity Laws	$A \cap U = A$	$A \cup \varnothing = A$
Annihilation Laws	$A \cup U = U$	$A \cap \varnothing = \varnothing$
Inverse/Complement Laws	$A \cup \overline{A} = U$	$A \cap \overline{A} = \varnothing$
Idempotent Laws	$A \cup A = A$	$A \cap A = A$
Commutative Laws	$A \cup B = B \cup A$	$A \cap B = B \cap A$
DeMorgans Laws	$\overline{(A \cup B)} = \overline{A} \cap \overline{B}$	$\overline{(A \cap B)} = \overline{A} \cup \overline{B}$
Absorption Laws	$A \cup (A \cap B) = A$	$A \cap (A \cup B) = A$
Associative Laws	$(A \cap B) \cap C = A \cap (B \cap C)$	$(A \cup B) \cup C = A \cup (B \cup C)$
Distributive Laws	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$