

MATH6055: *Provisional* Sample Test 2

15% Test Friday 30 November. Only material covered up to 20 November will be examined.

Name:

Answer all questions. Marks may be lost if necessary work is not clearly shown.

PLEASE READ ALL QUESTIONS CAREFULLY.

1. What is the output of the following Python code:

```
>>> a=6-4/2+5*2
>>> print a
```

2. Simplify

$$\frac{x^2 - 9}{x + 3}, \quad x \neq -3.$$

3. Write as a single fraction

$$x + \frac{1}{x + 5}, \quad x \neq -5.$$

4. Write as a single logarithm

$$\log_2(x^2) - \log_2(x) + 4 \log_2(\sqrt{x}), \quad x > 0.$$

5. Solve for x :

(a) $\frac{1}{x+4} + 3 = \frac{8}{x+5}$, **given that** $x \in \mathbb{Z}$.

(b) $x^2 + 7x = 0$.

(c) $\sqrt{x+13} = x+1$. Check your solution(s).

6. Solve for x :

(a) $y = \frac{x}{A} + B$.

(b) $s = ut + \frac{1}{2}xt^2$, $t \neq 0$.

7. Write in the form a^p , for $p \in \mathbb{Q}$:

$$\frac{a \cdot a^{-5}}{a^3}$$

8. Write in the form $a^p b^q$, with $p, q \in \mathbb{Q}$:

$$\left(\frac{a^6 b^5}{a^{-2} b^6} \right)^{-2}$$

9. Solve each of the following for x :

(a) $32^x + 4 = 20$,

(b) $\log_2(x^2 + 2x + 5) = 2$

(c) $e^x = 2^{1-x}$, **correct to four significant figures.**

10. Suppose you have an alphabet of size 62 and you want at least $10^9 = 1000000000$ possible passwords of the same length. What is the minimum length of password?

11. How many bits are required to represent $10^9 = 1000000000$ integers?

12. List the following list of functions $\mathbb{R}_+ \rightarrow \mathbb{R}$ from slowest growing to fastest growing:

$$3x - 2, \quad 90 \ln x, \quad e^{0.2x}, \quad x^{100}$$

Useful Formulae

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a^p a^q = a^{p+q}$$

$$\frac{a^p}{a^q} = a^{p-q}$$

$$(a^p)^q = a^{pq}$$

$$a^0 = 1$$

$$a^{-p} = \frac{1}{a^p}$$

$$a^{\frac{1}{q}} = \sqrt[q]{a}$$

$$a^{\frac{p}{q}} = \sqrt[q]{(a)^p} = (\sqrt[q]{a})^p$$

$$(ab)^p = a^p b^p$$

$$\left(\frac{a}{b}\right)^p = \frac{a^p}{b^p}$$

$$\log_a(xy) = \log_a x + \log_a y$$

$$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$$

$$\log_a(x^q) = q \log_a x$$

$$\log_a 1 = 0$$

$$\log_a\left(\frac{1}{x}\right) = -\log_a x$$

$$a^x = y \Leftrightarrow \log_a y = x$$

$$\log_a(a^x) = x$$

$$a^{\log_a x} = x$$

$$\log_b x = \frac{\log_a x}{\log_a b}$$