Antidifferentiation & Integration Exercises

Find each of the following (tables at the back of your manual):

1. $\displaystyle\int_2^7 (x^2 - 2x)\,dx$ **Ans:** $\dfrac{200}{3}$

2. $\displaystyle\int \left(4 - e^{-2x} + \sin x + \frac{3}{x}\right) dx$ **Ans:** $4x + \dfrac{1}{2}e^{-2x} - \cos x + 3\ln x + C.$

3. $\displaystyle\int 2x^2\sqrt{x^3+1}\,dx$ **Ans:** $\dfrac{4}{9}(x^3+1)^{3/2} + C$

4. $\displaystyle\int_1^{2-\sqrt{e}} \frac{1}{2-x}\,dx.$ **Ans:** $-\dfrac{1}{2}$

5. The work done by a spring in moving an object from a distance of $x = 0.1$ m from equilibrium to equilibrium at $x = 0$ m if the restoring force of the spring, in kilonewtons, is given by

$$F(x) = -80 \cdot x \qquad \textbf{Ans:} \quad -400 \text{ J}$$

6. $\displaystyle\int_0^{\pi} (1 + \sin x)\,dx$ **Ans:** $\pi + 2$

7. $\displaystyle\int_0^3 \frac{1}{\sqrt{9-x^2}}\,dx$ **Ans:** $\pi/2 \approx 1.57.$

8. $\displaystyle\int \sqrt{3x+4}\,dx$ **Ans:** $\dfrac{2}{9}(3x+4)^{3/2} + C$

9. $\displaystyle\int_0^{\pi/2} 3e^{\sin x}\cos x\,dx$ **Ans:** $3e - 1.$

10. The force F kN acting on a body is given by

$$F = 5x(1 - 0.001x^2);$$

where x is the displacement of the object in cm. Find the work done in moving the body from $x = 0$ cm to $x = 20$ cm. State your units. **Ans:** 8000 J.

11. $\displaystyle\int_1^{64} \left(\sqrt{t} - \frac{1}{\sqrt{t}} + \sqrt[3]{t}\right) dt$ $\left[\text{HINT: } \int t^n\,dt = \frac{t^{n+1}}{n+1}\right]$ **Ans:** $\dfrac{6215}{12}$

12. $\displaystyle\int (2 - \sqrt{x})^2\,dx.$ (Hint: multiply out) **Ans:** $4x - \dfrac{8}{3}x^{3/2} + \dfrac{1}{2}x^2 + C$

13. $\displaystyle\int (x^2+1)\sin(x^3+3x)\,dx$ **Ans:** $-\dfrac{1}{3}\cos(x^3+3x) + C$

14. $\displaystyle\int_e^{e^4} \frac{dx}{x\sqrt{\ln x}}.$ HINT: Again it's hard to see a function-derivative pattern but note:

$$\int \frac{dx}{x\sqrt{\ln x}} = \int \frac{1}{\sqrt{\ln x}} \times \frac{1}{x}\,dx,$$

Ans: 2.

15. A particle is moved along the x-axis by a force that measures $10/(1+x^2)$ N at a point x metres from the origin. Find the work done in moving the particle from the origin to a distance of 9 m.
Ans: $10\tan^{-1} 9 \approx 14.6$ J