Comments on MATH7019 Assignment 1 Submissions

1. *General Remarks*

   (a) **Nonsense Answers:** The number one thing I don't want to see is students giving answers that are *clearly* nonsense. Here is a selection.

   i. Student 2 thinks the below looks like a hanging chain:

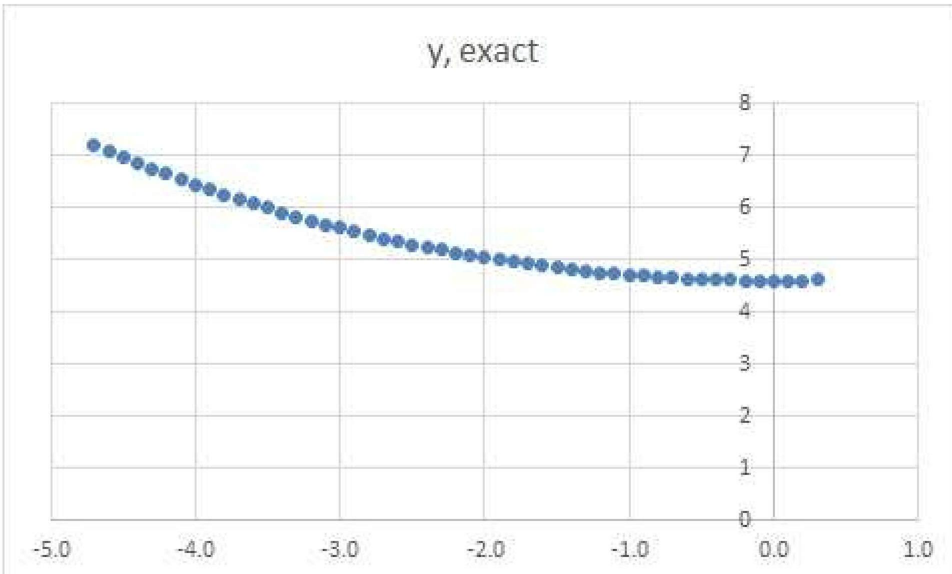

Figure 1: Funny-Looking-Chain.

   ii. Student 1 is trying to find an approximation to the chain on the left. He find the chain on the right and it doesn't seem to bother him that they look very different indeed.

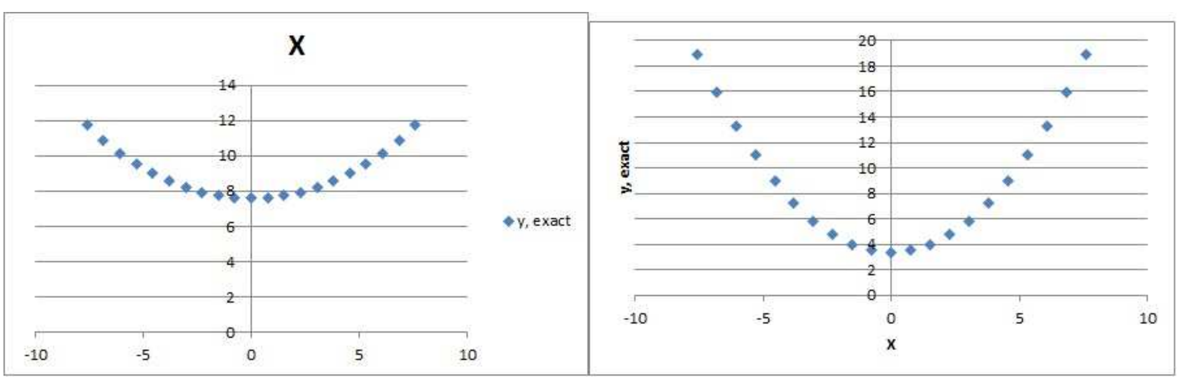

Figure 2: Note the second graph is very different to the first — at the boundaries the y-values are much different while at the minimum the y-values are also very different.

   iii. Student 2 is trying to extrapolate the maximum deflection of a beam of length 7 m. He comes up with an answer of 2.178 mm and doesn't realise this is clearly wrong as the beam of span 6 m already has a maximum deflection of 12.66 mm and so 2.178 mm is far too small.

iv. Student 5 is trying to find the grain size g such that half of the anthracite sample is smaller. Using his exponential model, the student gives me an answer of 19.6×10^8 mm. This is 1960 km... The student's data said that 17.33% of the sample had a grain size of 4.750 mm or less and 98.79% of the sample had a grain size of 9.525 mm or less... the answer should have been between these two.

(b) **Errors in Assignments:** A lot of us noted that things were wrong... yes by all means do this in an exam if you don't have time to fix the problem. However ye had plenty of time to do this assignment and so plenty of time to figure out problems... if you couldn't figure it out you should have asked for help. For example, Student 7 was trying to extrapolate the maximum deflection on a 7 m beam and came up with an answer of 2.326 mm. In fairness, the student pointed out that this couldn't be correct as the maximum deflection for a 6 m beam was 5.000 mm and so the answer for the 7 m beam had to be greater than this. I gave the student half a mark for this but he should have tried to fix the problem.

(c) **Wolfram Alpha:** When I wrote input

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D[a*Cosh[x/a],x] where x=a/2
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I meant *exactly* this except with your value of a. For example, with $a = 1.7$:

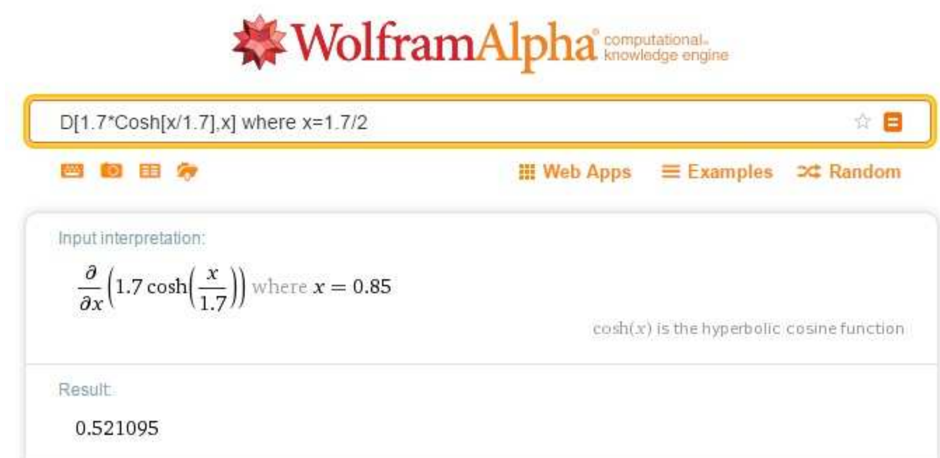

Figure 3: One student had problems with this and came to me. You could have had too. The answer is $m \approx 0.5211$.

2. *Mathematical Remarks*

(a) **Algebra:** Some of us are still making errors solving simple equations of the form:

$$ax + b = c.$$

Not good enough.

Solve $30x - 3 = 3$

The one thing we never do here yet it is at the heart of the whole problem is to assume that x *solves the equation. Now we can proceed as follows (note that when we are more proficient we can safely skip steps here). The 'method' is that you identify what is stopping* x *from being on its own (we want to end up with* $x = \dots$ *after all), and say what you need to do to get rid of what is in the way. Now, simply, do what you need to do and — critically —* do it to both sides of the equation. *For if* $5 = 5$ *then* $5 + 3 = 5 + 3$*, etc. — by doing the same thing to both sides the equality is preserved*[1]*.*

In this example, $30x - 3 = 3$ *what is annoying me firstly is the* -3 *on the LHS. I can get rid of* -3 *by adding* 3 *to both sides and I could write:*

$$\begin{aligned} 30x - 3 + 3 &= 3 + 3 \\ \Rightarrow 30x + 0 &= 6 \\ \Rightarrow 30x &= 6. \end{aligned}$$

What we have done here is started with an equation — which is a true fact if x *is the solution — and done the same thing to both sides of the equation to generate another true fact... the* $\Rightarrow$ *here means "means that" or "it implies that".*

Now the 30 *is in the way. What I can do here is divide both sides by* 30*. In fact there is no need for division at all — instead of dividing by* 30 *I can multiply by* $\dfrac{1}{30}$ *— and as long as I do it to both sides I generate a new true fact about* x*:*

$$\begin{aligned} \frac{1}{30} \cdot 30x &= \frac{1}{30} \cdot 6 \\ \Rightarrow \frac{30}{30} x &= \frac{6}{30} \\ \Rightarrow 1 \cdot x &= \frac{6}{30} \cdot \underbrace{\frac{\frac{1}{6}}{\frac{1}{6}}}_{=1} \\ \Rightarrow x &= \frac{1}{5}. \end{aligned}$$

where again I took a load more steps than necessary just to show what is really happening.

Note the whole way through there is no 'moving' of numbers... that is what it looks like but is not what is going on. The students who got this wrong were 'moving numbers' rather than deriving true facts about x *until we arrive at the ultimate true fact about* x *— what it is equal to as a real number.*

[1] with a few subtleties

(b) **The Line:** It is on p. 83 of the notes. If y and x are related according to

$$y = mx + c$$

and we plot y vs x we get a straight line graph of slope m and y-intercept c. Saying m and/or c are constants tells me precisely nothing about the graph of y vs x.

(c) **Percentage Error:** By convention, percentage error is given by:

$$\text{percentage error} = \frac{\text{error}}{\text{true value}}. \tag{1}$$

For example, if the true value of the slope of the chain at a certain point is ≈ 0.5211, and the parabolic approximation is ≈ 0.5358, then the percentage error in this approximation is

$$\text{percentage error} = \frac{\Delta Q}{Q_0} = \frac{|0.5358 - 0.5211|}{0.5211} \approx 0.02821 = 2.821\%.$$

(d) **Slope of a Curve:** The slope of (the tangent to) a curve is given by the derivative. This is first year material. The derivative gives you the slope of the tangent to the curve at any point x. Say, for example, that you had a (hanging-chain-approximating) curve

$$y = 0.141x^2 + 3.788.$$

Then the slope at x is equal to

$$\frac{dy}{dx} = 0.141(2x) + 0 = 0.282x.$$

We were interested in the slope of the chain at $x = a/2$, which for this chain above, was $a/2 = 1.9$. So $\frac{dy}{dx}$ gives the slope at any point x... and we are interested in $x = 1.9$:

$$\text{Ans: slope at } x = 1.9 = \left.\frac{dy}{dx}\right|_{x=1.9} = 0.282x|_{x=1.9} = 0.282(1.9) = 0.5358.$$

(e) $y(x)$ **Notation:** When talking about a variable y depending on another variable x, for example

i. the height of the chain y depends on the position x
ii. the maximum deflection δ depends on the length of the beam L and
iii. the passing percentage P depends on the grain size g
iv. the biofuel consumption C depends on time t

we say that y is a function of x and we write $y = f(x)$ or $y = y(x)$. This does *not* mean multiplication. For example,

i. writing $y = y(x)$ just means y depends on x... for each value of x there is only one corresponding value of y.
ii. $\delta(L) = a \cdot L^N$ does *not* mean $\delta \times L$ but that δ depends on L, δ is a function of L.
iii. $P(g) = A \cdot e^{kg}$ means that P depends on g. Just dwelling on this, some students wanted to find the g such that $P = 50$ but substituted $g = 50$... no. It was $P = P(g) = 50$ so you had to solve:

$$50 = A \cdot e^{kg},$$

where you had A and k from your log-linear least squares and you had to find the corresponding g.
iv. $C(t) = m \cdot t + c$ does not mean "consumption times time"

3. *Curve-Fitting Remarks*

(a) $\sum c = Nc \neq (N-1)c$**:** So many students, for some unknown reason, took $\sum c = 17c$ if there were, for example, 18 data points. No. If there are N data points, $\sum c = Nc$. Always.

(b) **Fitting only to the Exponential Data:** I gave you 'S' shaped data:

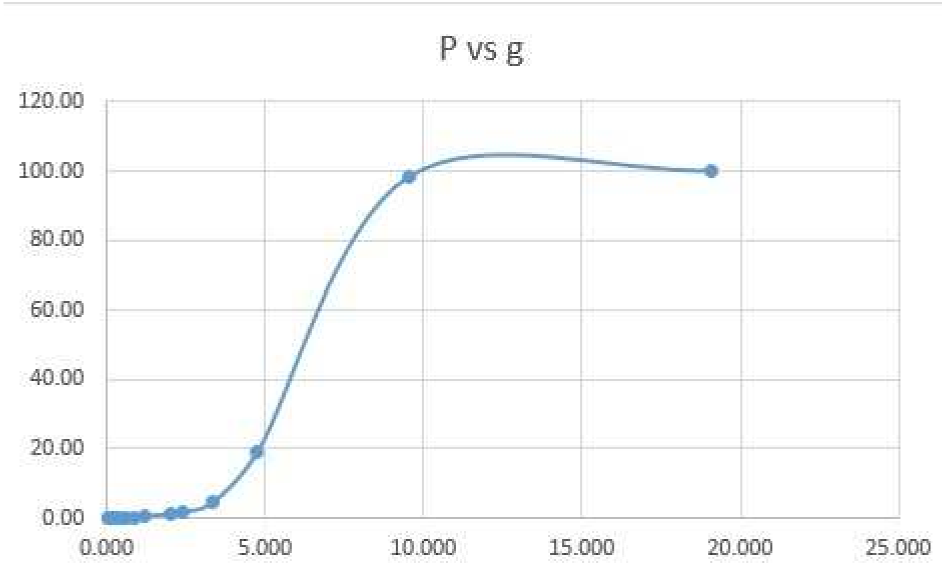

Figure 4: The entirety of the P vs g Data.

There is no point fitting an exponential to this because it looks nothing like an exponential. If you do fit an exponential to it you get:

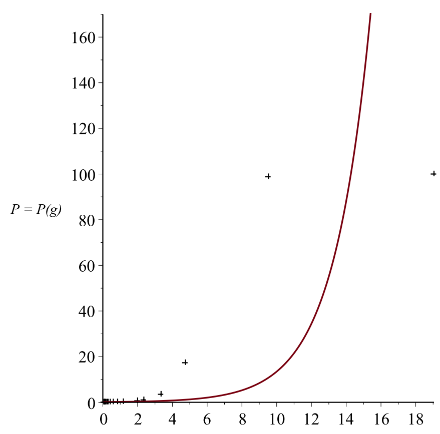

Figure 5: This isn't a good fit to the 'S' data above and predicts that half the sample has a grain size of 12.8 mm or less... which is nonsense because nearly 100% of the sample size has a grain size of about ten or less.

Instead either the $P = 100$ or both the $P = 100$ and $P \approx 100$ data points should have been thrown out and just the following, *relevant data* should have been looked at:

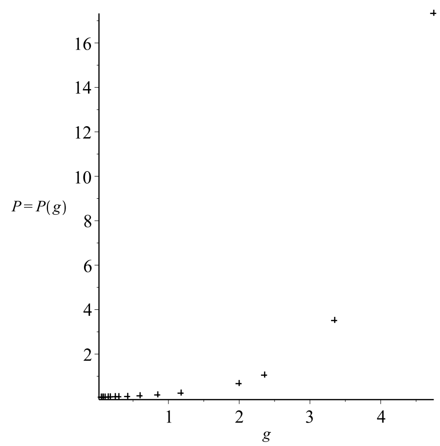

Figure 6: This data looks a lot like an exponential — we threw out the $P = 100$ and $P \approx 100$ data points.

Fitting an exponential to this yields $g = 5.56$ mm:

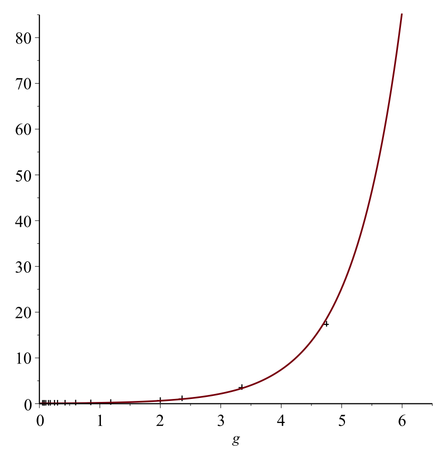

Figure 7: This is a much better fit to the data.

An even more drastic problem is the following. We start with $P = A \cdot e^{kg}$ and we linearise to:

$$\ln P = k \cdot g + \ln A.$$

If P and g are related as $P = A \cdot e^{kg}$, then $\ln P$ and g are related by the above. Note the above has the form of a line:

$$y = m \cdot x + c.$$

Therefore if we plot $\ln P$ vs g, for the exponential data, we should get a straight line.

If a student tries to fit a straight-line to the exponential, untransformed, (g, P) data... well it doesn't make much sense:

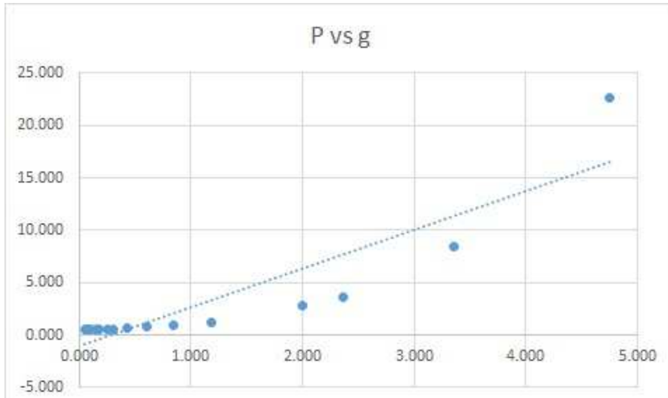

Figure 8: We aren't fitting a straight-line to the (g, P) data.

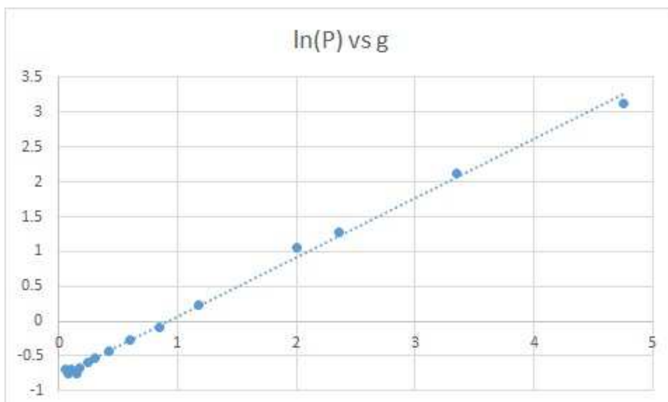

Figure 9: Here we see a straight line fit to $(g, \ln P)$ data. In the previous part, you showed that $P = Ae^{kg}$ could be rewritten as $y = mx + c$ with $y = \ln P$ and $x = g$. Therefore use $x = g$ and $y = \ln P$ not the original, untransformed data.

A similar issue occurred with Question 2. There isn't a straight-line relationship between δ and the length of the beam, but there is a straight-line relationship between $\ln(\delta)$ and $\ln(L)$.

(c) **Outliers** In the exam, Q. 4, there won't be any *outliers*: data points that are untypical of the rest of the data. However, in this assignment, some of the data had outliers that could be disregarded. I wasn't docking marks for this but no harm in pointing it out. Student 4 plotted the Biofuel consumption of a country/region and saw this:

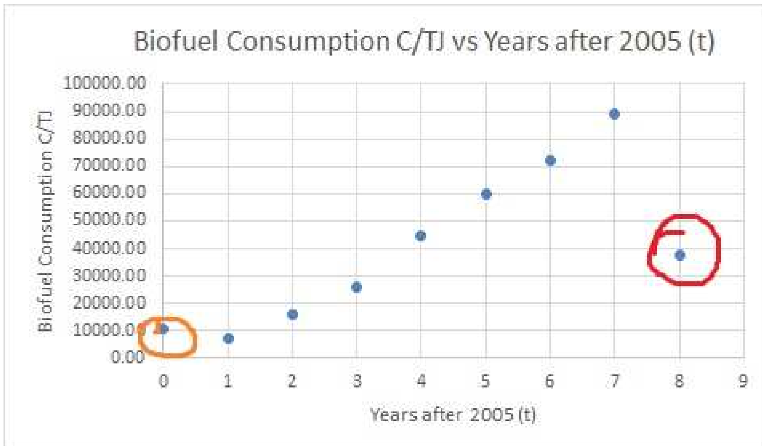

Figure 10: The red data point is untypical. Perhaps there was a data entry error. Perhaps this was only for half of 2013. Either way the student could have thrown this data point out. The orange data point might have to be looked at too.