MATH7021: Assignment 1 — Remarks for Poorly Answered Questions

April 9, 2018

1 General Remarks

- given that ye had the time, it would have been a good idea to check your answers by substitution. This would have saved a lot of us lost marks. It is very easy to solve *one* of the equations (with incorrect values) — so if you are checking, check that *all* equations are satisfied.
- Note that if you are doing Gaussian Elimination by hand you should use exact fractions and square roots rather a decimal approximation. If you are using rounding when doing Gaussian Elimination you have to do partial pivoting.
- Let me be clear because some of this didn't get this:
  - Gaussian Elimination is very senstive[1] to ROUNDING ERROR, therefore
  - when you are doing Gaussian Elimination you may NOT use rounding. However,
  - if you *must* use rounding (such as when you are on a computer; i.e. Excel), you MUST DO PARTIAL PIVOTING.

2 2×2 Linear Systems

Remark: A number of us made a serious logical error here... take part (a). We need to prove the result for *ALL* systems such that $m_1 \neq m_2$. Showing the result for a single example, say $m_1 = 3$ and $m_2 = 5$, isn't sufficient to prove the result for all systems such that $m_1 \neq m_2$.

For example, suppose you were asked to prove the following 'theorem':

> *All Brazilians are good at soccer.*

Pointing out one Brazilian is good is not sufficient to prove that *all* Brazilians are good.

[1] what this means is that if you round you can get very different solutions

2.1 Illegal 'Row-Operations'

Consider a 2×2 linear system given by

$$\begin{aligned} x + 2y &= c_1 \\ 3x + 4y &= c_1 + 10 \end{aligned}$$

Written in augmented matrix form this is given by:

$$\left[\begin{array}{cc|c} 1 & 2 & c_1 \\ 3 & 4 & c_1 + 10 \end{array}\right]. \tag{1}$$

(a) Suppose we do the 'row operation' $r_2 \to r_2 \times r_1$:

$$\left[\begin{array}{cc|c} 1 & 2 & c_1 \\ 3 & 8 & c_1^2 + 10c_1 \end{array}\right].$$

This corresponds to the linear system:

$$\begin{aligned} x + 2y &= c_1 \\ 3x + 8y &= c_1^2 + 10c_1 \end{aligned} \tag{2}$$

iii. What can you conclude about 'row operations' of the form $r_i \to r_i \times r_j$?

Remark: Some of us didn't notice that the solution to linear system (2) was different to the solution to linear system (1). Therefore you can't multiply rows together when doing Gaussian Elimination. Similarly for part (b).

3 Gaussian Elimination: Abstract Problems

(a) * Using three-decimal-place rounding, use *Microsoft Excel* to solve the linear system:

$$\begin{pmatrix} 1.80 & 2.52 & 4.50 \\ 1.60 & 5.12 & 5.44 \\ 2.50 & 6.50 & 9.25 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}$$

Remark: Gaussian Elimination without partial pivoting is very susceptible to rounding error. Therefore if you are rounding when doing Gaussian elimination you *must* use partial pivoting.

Also it isn't sufficient just to 'pivot' the first column alone — if the second column needs it you must pivot a second time. For example, after pivoting once, making the one and then the zeroes underneath, most of us got to here:

$$\left[\begin{array}{ccc|c} 1 & 2.6 & 3.7 & 0.4c_3 \\ 0 & 0.96 & -0.48 & c_2 - 0.64c_3 \\ 0 & -2.16 & -2.16 & c_1 - 1.8c_3 \end{array}\right]$$

As -2.16 is bigger than 0.96 in magnitude the next row operation should have been $r_2 \leftrightarrow r_3$ not $r_2 \to r_2/0.96$ as most of us did.

Remark: Some people did the Gaussian Elimination but never found a solution:

$$
\begin{aligned}
w &= \cdots \\
\Rightarrow z &= \cdots \\
\Rightarrow y &= \cdots \\
\Rightarrow x &= \cdots
\end{aligned}
$$

3.1 A Traffic Flow Problem

Consider the following traffic flow problem:

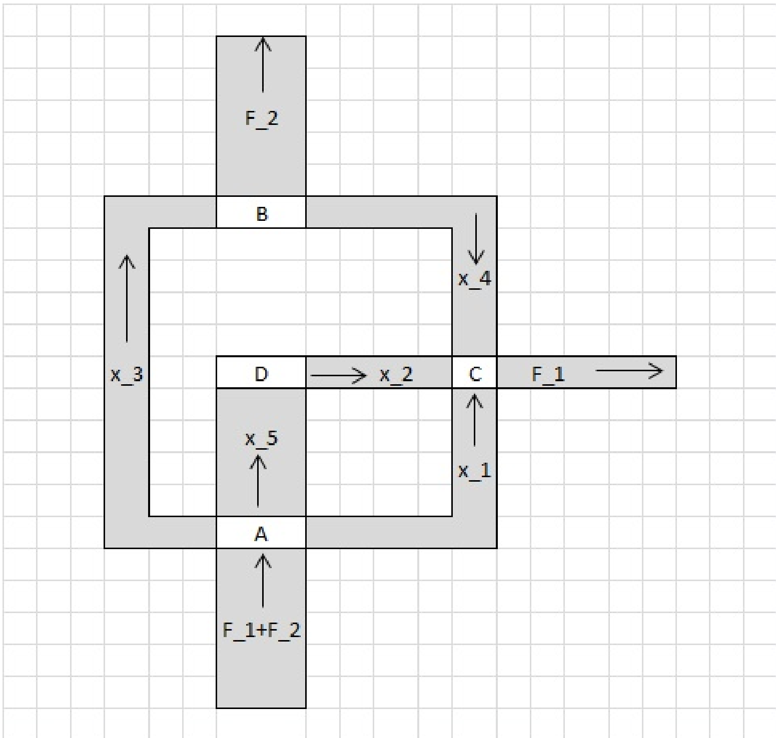

Figure 1: There are three different routes for those going from South at A to East at C. Take $F_1 = 400 + 40c_1$ and $F_2 = 10 + 10c_2$

(c) If you assume that the speed of traffic is inversely proportional to the traffic flow, then the time taken to travel a route is given by:

$$\text{time} = \text{distance} \times \text{traffic flow}.$$

Take the following distances:

route	distance
$A \to B$	15
$B \to C$	9
$A \to D \to C$	9
$A \to C$ direct	9

If the time taken to travel from $i \to j \to k$ is the same as the time taken to travel $i \to j$ *plus* the time taken to travel $j \to k$, write, in terms of your parameters, expressions for the time taken to travel:

i. $T_1 := A \to B \to C$

ii. $T_2 := A \to D \to C$

iii. $T_3 := A \to C$ direct

Remark: The biggest problem here was T_2 and some people made the same mistake. The thing is that the speed from $A \to D$ was equal to the speed from $D \to C$, namely $x_5 = x_2 = t$. Therefore the flow over this distance of nine was t so

$$T_2 = 9t,$$

rather than $18t$ as some people had.

3.2 Truss Systems: the Method of Joints

(d) Complete the following. I use the word *suggests* because things aren't always as straightforward as the above three examples.

Remark: I agree that the first part is hard on its own however if you go through the three points that suggests what the correct words for the first part were.

- *A truss system composed of b BEAMS and r REACTIONS subject to an external FORCE has $b + r$ VARIABLES.*

  Using the method of JOINTS, for each JOINT in a truss system we can write down two EQUATIONS. Therefore if there are j joints in the truss system the associated linear system has $2j$ EQUATIONS.

- *If $b+r = 2j$, then the number of VARIABLES equals the number of EQUATIONS. This suggests that when the linear system is brought into row-reduced form, there are no COLUMNS without PIVOTS. Therefore there are no parameters, and hence a UNIQUE solution. Therefore the truss system is determinate.*

- *If $b + r > 2j$ then the number of VARIABLES is greater than the number of EQUATIONS. This suggests that when the linear system is brought into row-reduced form there are COLUMNS without PIVOTS. In this case we have at least one PARAMETER so there are an INFINITE number of solutions. In this case the truss system is said to be indeterminate.*

- *If $b + r < 2j$ then the number of VARIABLES is less than the number of EQUATIONS so that when the linear system is brought into row-reduced form there is a chance that there are NO solutions. In this case the assumption that the truss system is STABLE leads to an absurdity so that we must conclude that the truss system is UNSTABLE.*

4 Heat Distribution Problem

Do you think you are close to the solution of the linear system? Justify your answer.

Remark: When doing the Jacobi Method you can't be sure you are close to the solution until you have universal agreement between all x_n^i and x_n^{i+1} to so many decimal places. In this question all of us had a lot of fluctuation between the first and second iterations so none of us were close to the solution.