1. (a) When finding the partial fraction expansion of a Laplace Transform $F(s)$, the following simultaneous equations had to be solved:

$$A + B + C + D = 3$$
$$2A - 4B + 5C - D = -9$$
$$4A + 4B - 2C - 2D = 3$$
$$6A - 4B + 7C + 3D = -4$$

Use *Gaussian elimination* to find the solution of the linear system.

[8 Marks]

(b) Use *Gaussian elimination* to solve the linear system:

$$x + 2y - z = 2$$
$$2x + 5y + 2z = -1$$
$$7x + 17y + 5z = -1$$

[5 Marks]

(c) Consider the following traffic flow diagram:

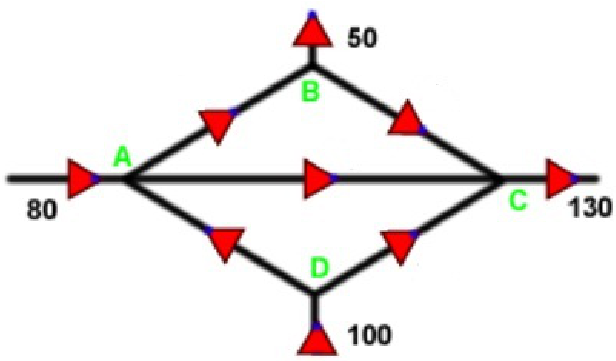



Figure 1: A Traffic Flow System

i. Write down the linear system governing the flow.

[3 Marks]

ii. Use *Gaussian elimination* to solve the linear system.

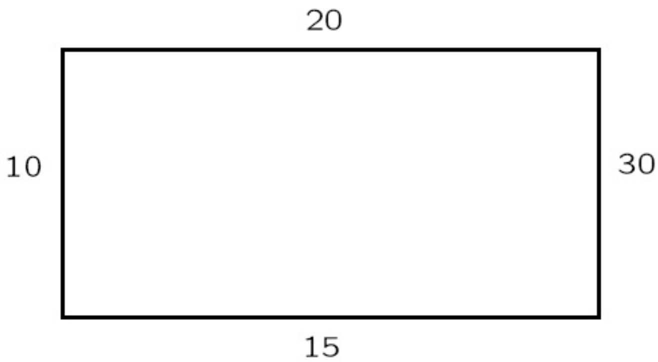
[7 Marks]

(d) Use Cramer's Rule to solve the linear system

$$\begin{aligned} 4x + 5y &= 8 \\ x - y &= 11 \end{aligned}$$

[3 Marks]

(e) The following plate has dimensions 4:2 and is subject to boundary temperatures as shown:




i. Using an appropriate grid, find a linear system whose solution approximates the heat distribution of the plate at internal grid-points using the *Mean-Value Property*.

[3 Marks]

ii. Use *Gaussian elimination with partial pivoting* to solve the linear system. Use two places of decimals for all calculations.

[6 Marks]

2. (a) Find, using **only** the method of undetermined coefficients, the general solution of the ordinary differential equation

$$\frac{d^2x}{dt^2} - \frac{dx}{dt} - 6x(t) = \cos t.$$

[7 Marks]

(b) Solve, using **only** the method of undetermined coefficients, the initial value problem

$$\frac{d^2x}{d\theta^2} + 4x(\theta) = 2\theta\,; \qquad x(0) = 1\,,\ x'(0) = 2.$$

[8 Marks]

3. (a) Solve using **only** the *Laplace Transform Method* for $x = x(t)$,

$$4\frac{dx}{dt} + x(t) = 20\,, \;\; x(0) = 0.$$

[8 Marks]

(b) The differential equation governing the displacement $x(t)$ of a *damped harmonic oscillator* is given by:

$$y''(t) + 2y'(t) + 37y(t) = 0,$$

with the initial conditions $x(0) = 0$ and $y'(0) = 3$.

i. Solve the differential equation using **only** *Laplace transforms.*

[9 Marks]

ii. Is the oscillator over- or under-damped?

[1 Mark]

(c) The temperature in $^\circ C$, $\theta(t)$, of a metal at a time $t$ s after being immersed at a temperature of $\theta(0) = 300\,^\circ C$ in a reservoir of temperature $50\,^\circ C$ is given by

$$\frac{d\theta}{dt} = -10(\theta(t) - 50).$$

i. Use **only** *Laplace Methods* to solve the differential equation for the temperature $\theta(t)$ at any time $t$. An appropriate notation for $\mathcal{L}\{\theta(t)\}$ is $T(s)$.

[5 Marks]

ii. Find the time, $t$, such that $\theta(t) = 100$.

[2 Marks]

(d) Solve the following system of differential equations using *Laplace Transforms*:

$$\frac{dx}{dt} = 6x(t) - 3y(t)\,, \;\; x(0) = 0$$

$$\frac{dy}{dt} = -2x(t) + y(t)\,, \;\; y(0) = 1$$

[10 Marks]

4. (a) A triangular region has vertices $(0,0)$, $(2,0)$ and $(0,1)$. Find the second moment of area of this region about the $x$-axis:

$$I_{xx} = \iint y^2\,dA.$$

[7 Marks]

(b) A cylinder described by $x^2 + y^2 \leq 4$ and $0 \leq z \leq 3$ has density $\rho(x,y,z) = 2y^2z$. Find the mass of the cylinder:

$$m = \iiint_V \rho(x,y,z)\,dV.$$

[HINT: $\sin^2 A = \frac{1}{2}(1 - \cos 2A)$]

[8 Marks]