CORK INSTITUTE OF TECHNOLOGY
INSTITIÚID TEICNEOLAÍOCHTA CHORCAÍ

Autumn Examinations 2012/13

Module Title: Technological Mathematics 312

Module Code:	MATH 7021
School:	Building & Civil Engineering
Programme Title:	Bachelor of Engineering in Civil Engineering – Year 3
Programme Code:	CCIVL_7_Y3
External Examiner(s):	Mr. C. O'Sullivan
Internal Examiner(s):	Mr. J.P. Mc Carthy
Instructions:	Answer four questions. All questions carry equal marks
Duration:	2 Hours
Sitting:	Autumn 2013
Exam Requirements:	Mathematics Tables

Note to Candidates: Please check the Programme Title and the Module Title to ensure that you are attempting the correct examination.
If in doubt please contact an Invigilator.

1. (a) It is believed that variables Y and t have a linear relationship of the form

$$Y = At + B$$

for constants A and B. For the data below find the line of best fit *in the least squares sense* and hence write down the values of A and B.

t	1	2	3	4	5	6
Y	-1	2	5	8	11	13

(7 marks)

(b) Carry out *Gaussian Elimination* on the following linear system until it is in reduced row form. Do **not** use decimal rounding.

$$\begin{pmatrix} 3 & -1 & 1 \\ 1 & 1 & -1 \\ 2 & -2 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \\ -1 \end{pmatrix}.$$

(5 marks)

Show that there are an infinite number of solutions and find these solutions in terms of a parameter t.

(4 marks)

(c) Suppose that the number of moles $n(t)$ of a radioactively decaying material satisfies the first order differential equation:

$$\frac{dn}{dt} = -0.01n(t).$$

Use *Laplace Methods* to solve the differential equation for the number of moles $n(t)$ at any time t if there are initially ten moles: $n(0) = 10$.

(9 marks)

2. (a) Corresponding values of x and y for a polynomial function are given in a table attached to this examination paper. There is an error in the table. Form a *forward difference* table up as far as and including second differences for these values. Include the completed table with your answer sheet.

   i. Locate and correct the error in the table.

   (5 marks)

   ii. Extend the table to estimate the values of y at $x = 0$ and $x = 120$.

   (2 marks)

   iii. Use *linear interpolation* to estimate the value of y at $x = 53$.

   (3 marks)

   iv. Use the *Newton-Gregory Interpolation formula* to approximate the value of y at $x = 86$.

   (3 marks)

   v. Estimate the value of $y'(x)$ at $x = 26$.

   (2 marks)

   (b) The following times t and outputs θ were measured and recorded:

input, t	0	5	10	15	20
output, θ	58	52	47	42	37

   It is believed that t and θ have a relationship of the form:

$$\theta = Ae^{-kt}.$$

   i. Find the best values of A and k in the *least squares sense*. Use three places of decimals in all calculations.

   (7 marks)

   ii. Also, use the data at times $t = 10$, 15 and 20 to estimate the value of θ at $t = 12$ using *Lagrangian Interpolation*.

   (3 marks)

3. (a) Use *Gaussian Elimination* to find the solutions of the simultaneous equations

$$\begin{aligned} x + y + z - w &= 0 \\ 4x - y + 2z + 3w &= -2 \\ 4x + y + 2z + w &= 2 \\ 3x + 2y + 4z + 5w &= -6 \end{aligned}$$

[HINT: Do **not** use decimal rounding]

(7 marks)

(b) Use *Gaussian Elimination with Partial Pivoting* to estimate, **correct to two decimal places**, the solution set of the linear system:

$$\begin{pmatrix} 1 & 1 & 1 \\ 2 & 3 & 1 \\ 4 & 8 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ 11 \\ 23 \end{pmatrix}.$$

(7 marks)

(c) Consider the set of simultaneous equations:

$$\begin{aligned} 6x - y &= 24 \\ x + 10y &= -4 \end{aligned}$$

By replacing this linear system by a simpler one, find a first approximation (x_0, y_0) to the solution.

(1 mark)

Use **three iterations** of *Jacobi's Method* **AND two iterations** of the *Gauss-Siedel Method* to estimate **correct to two decimal places** the solution of simultaneous equations.

(7 marks)

Use *Cramer's Rule* to find the exact values of x and y and compare with your estimates from using *Jacobi's Method* and the *Gauss-Siedel Method.*

(3 marks)

4. Use *Laplace Methods* to solve the following differential equations

   (a) $\dfrac{dx}{dt} + 16x(t) = 64 \qquad x(0) = 1.$

   (7 marks)

   (b) $2\dfrac{d^2x}{dt^2} + 4\dfrac{dx}{dt} + 20x(t) = 0 \qquad x(0) = 4,\ x'(0) = -4.$

   (9 marks)

   (c) $\dfrac{d^2y}{dx^2} - 4\dfrac{dy}{dx} + 4y(x) = 24e^{4x} \qquad y(0) = y'(0) = 0.$

   (9 marks)

5. (a) Sketch the triangular region with vertices $(-1, 0)$, $(1, 0)$ and $(0, 4)$.

   (1 mark)

   i. Evaluate the *line integral*

   $$\oint_C (6x^2\,dx + 12y^2\,dy)$$

   where C is the perimeter of this region.

   (6 marks)

   ii. By evaluating an appropriate double integral, find the *second moment of area* of this region about the x-axis.

   (6 marks)

   (b) A cylinder is described by

   $$x^2 + y^2 \leq 9 \quad \text{and} \quad 0 \leq z \leq 2.$$

   i. Evaluate the *line integral*

   $$\oint_C (12xy\,dx + 24y^2\,dy),$$

   where C is the perimeter of the base of the cylinder.

   (6 marks)

   ii. For this volume, evaluate the *triple integral*

   $$\iiint_V x^2 z\,dV.$$

   (6 marks)

$$\left[\text{HINT: } \cos^2 A = \frac{1}{2}(1 + \cos 2A) \text{ and } \sin^2 A = \frac{1}{2}(1 - \cos 2A)\right]$$

LAPLACE TRANSFORMS-*

For a function f(t) the Laplace Transform of f(t) is a function in s defined by

$$F(s) = \int_0^\infty e^{-st} f(t)dt \quad \text{where } s>0.$$

f(t)	F(s)
A=constant	$\frac{A}{s}$
t^N	$\frac{N!}{s^{N+1}}$
e^{at}	$\frac{1}{s-a}$
sinhkt	$\frac{k}{s^2-k^2}$
coshkt	$\frac{s}{s^2-k^2}$
$\sin\omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos\omega t$	$\frac{s}{s^2+\omega^2}$
$e^{at} f(t)$	F(s-a)
$f'(t)$	sF(s)-f(0)
$f''(t)$	$s^2F(s) - sf(0) - f'(o)$

Note: $\cosh A = \frac{e^A + e^{-A}}{2}$ $\qquad$ $\sinh A = \frac{e^A - e^{-A}}{2}$

DERIVATIVES

f(x) a=constant	f′(x)
x^n	nx^{n-1}
lnx	$\frac{1}{x}$
e^{ax}	$a e^{ax}$
sinx	cosx
cosx	-sinx

INTEGRALS

f(x) a=constant	$\int f(x)dx$
x^n	$\frac{x^{n+1}}{n+1}$ if $n \neq -1$
$\frac{1}{x}$	lnx
e^{ax}	$\frac{1}{a} a e^{ax}$
sinx	-cosx
cosx	sinx

INTERPOLATION

$$f(x) = \frac{(x - x_1)(x - x_2)}{(x_0 - x_1)(x_0 - x_2)} f(x_0) + \frac{(x - x_0)(x - x_2)}{(x_1 - x_0)(x_1 - x_2)} f(x_1) + \frac{(x - x_0)(x - x_1)}{(x_2 - x_0)(x_2 - x_1)} f(x_2)$$

$$f(x_0 + rh) = f(x_0) + r\Delta f_0 + \frac{r(r - 1)}{2!} \Delta^2 f_0 + \ldots$$

$$f'(x_0 + rh) = \frac{1}{h}\left[\Delta f_0 + \frac{(2r - 1)}{2!} \Delta^2 f_0 + \ldots\right]$$

x	y	Δy	$\Delta^2 y$
0			
10	1		
20	3		
30	7		
40	13		
50	21		
60	33		
70	43		
80	57		
90	73		
100	91		
110	111		
120			