

MATH7021: Sample Test 2

Name:

Student Number:

Answer all questions. Marks may be lost if necessary work is not clearly shown.

1. Find the *Laplace Transform of the Solution* of each of the following differential equations. Do not evaluate any partial fractions or Inverse Laplace Transforms.

(a) $\frac{d^2x}{dt^2} - x(t) = 2e^{3t} \quad x(0) = 1.$

[4 Marks]

(b) $x''(t) + 2x'(t) + 10x(t) = 0; \quad x(0) = 0, x'(0) = 6.$

[4 Marks]

(c) $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y(x) = 80 \cos 2x; \quad y(0) = y'(0) = 0.$

[4 Marks]

[HINT: i.e. in each case find ' $F(s)$ ']

2. Do not evaluate any Inverse Laplace Transforms.

(a) Find the *partial fraction expansion* of $\frac{80}{s^2 - 7s + 10}.$

[2 Marks]

(b) Find the *partial fraction expansion* of $\frac{s + 2}{(s^2 + 9)(s - 2)(s + 1)}.$

[5 Marks]

- (c) By first *completing the square*, write, for constants A, B, a, ω

$$\frac{2s - 6}{s^2 - 2s + 17} = A \cdot \frac{s - a}{(s - a)^2 + \omega^2} + B \cdot \frac{\omega}{(s - a)^2 + \omega^2}.$$

[4 Marks]

3. Find the *Inverse Laplace Transforms* of the following:

(a) $F(s) = \frac{1}{s} + \frac{3}{s - \frac{1}{3}}$.

[2 Marks]

(b) $G(s) = 4\frac{s - 3}{(s - 3)^2 + 9} - 6\frac{3}{(s - 3)^2 + 9}$

[3 Marks]

(c) $H(s) = \frac{4s}{s^2 + 1} - \frac{1}{s^2 + 1} + \frac{3}{(s - 1)^2}$

[3 Marks]

4. Suppose that the charge $q(t)$ on a capacitor satisfies the first order differential equation:

$$\frac{dq}{dt} + 2q(t) = 2.$$

Use *Laplace Methods* to solve the differential equation for the charge on the capacitor plate $q(t)$ at any time t if the plate is initially uncharged $q(0) = 0$.

[8 Marks]