MATH6038: Chapter One Matrix Algebra Test & Exam Questions

Note that your test will comprise of approximately 30 marks of questions *like* from this list here.

Useful Formula: $A^{-1} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{\det A} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$ where $\det A = ad - bc$.

1. Use Gaussian Elimination to determine the solution set S for each of the following systems of linear equations. Clearly describe the solution set S in each of the three cases.

$$(A)\ \begin{array}{rcrcl} x & + & 3y & = & 4 \\ 4x & + & 12y & = & 17 \end{array} \qquad (B):\ \begin{array}{rcrcl} x & + & 2y & = & 3 \\ 2x & + & 4y & = & 6 \end{array} \qquad (C):\ \begin{array}{rcrcl} x & + & 3y & = & 2 \\ 4x & + & 18y & = & 16 \end{array}.$$

[13 Marks]

2. Given the following row reduced augmented matrix, write down the associated linear system of equations in terms of the variables x_1, x_2, x_3 and x_4. Identifying the free variable and express the solutions set in terms of the parameter t.

$$\left[\begin{array}{cccc|c} 1 & 0 & 4 & 0 & 5 \\ 0 & 1 & 9 & 0 & 3 \\ 0 & 0 & 0 & 1 & 8 \end{array}\right].$$

[6 Marks]

3. A secondary school wants to lease 18 buses with a combined carrying capacity of 990 students. The three available types of buses carry 20, 35 and 65 passengers, respectively.

   (a) Identify variables x, y and z.

   (b) Write down the corresponding *linear system* and find the system's *solution set* $\mathcal{S}$ including the real parameter $t \in \mathbb{R}$.

   (c) Find out how many of each type of bus could be leased by finding all *positive* solutions.

[15 Marks]

4. Use Gauss-Jordan elimination to find A^{-1} where

$$A = \begin{bmatrix} 1 & 0 & 8 \\ 2 & 5 & 3 \\ 1 & 2 & 3 \end{bmatrix}.$$

[10 Marks]

5. Using row reduction, or otherwise, find A^{-1} where $A = \begin{bmatrix} -2 & 4 \\ 6 & -1 \end{bmatrix}$.

Hence solve the matrix equation for x and y:

$$\begin{bmatrix} -2 & 4 \\ 6 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 16 \\ -10 \end{bmatrix}.$$

[9 marks]

6. Apply *only Cramer's Rule* to solve the system of linear equations **just for variable** y,

$$\begin{aligned} x + y + 2z &= -2 \\ 3x + 6y - 5z &= 8 \\ 2x + 4y - 3z &= 10. \end{aligned}$$

[7 marks]

7. The system of linear equations given by

$$\begin{aligned} x - 2y + 4z &= 1 \\ -x + 3y + z &= -2 \\ 2x - 5y + 2z &= 4, \end{aligned}$$

can be expressed by the matrix equation $AX = B$. Identify the coefficient matrix A and the column matrices X and B. Determine A^{-1} and hence solve the above system. Verify your answer for z

[14 Marks]

8. Use only *determinants* to determine if the following homogenous system of linear equations has either the trivial solution *or* non-trivial solutions.

$$\begin{aligned} 2x - 4y - 5z &= 0 \\ 3x + y - 4z &= 0 \\ x - 6y - z &= 0. \end{aligned}$$

[7 Marks]