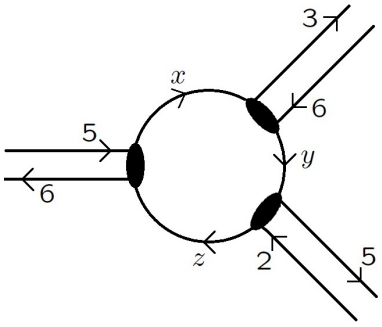
1. (a) When a rotary device is used for 10 minutes, its temperature, $\theta(\omega)$, depends on the angular frequency, ω. The following recordings were made

angular frequency, ω/ Hz	1	2	3	4	5
temperature, $\theta/^\circ$ C	31	42	59	68	79

Use one of the interpolation methods developed in lectures to estimate $\theta(2.4)$. [6 marks]

(b) Consider the following traffic flow diagram.



Using the principle that the flow into a junction equals the flow out, it can be shown that the equations governing the traffic flow are given by

$$\begin{aligned} x - y &= -3 \\ y - z &= 3 \\ x - z &= -1 \end{aligned}$$

Write these equations in *augmented matrix form* and solve using *Gaussian elimination.* [6 Marks]

Interpret your solution in terms of the traffic flow diagram. [1 Mark]

(c) The deflection, $y(x)$, of a beam at a point x m along the beam satisfies

$$\frac{d^2y}{dx^2} = \frac{5}{6}x^2 - 6x + 9 \; ; \qquad y(0) = y'(0) = 0.$$

Use *Laplace methods* to solve for $y(x)$. [6 Marks]

(d) By direct calculation, or otherwise, show that when $\mathcal{R} : x^2 + y^2 = 1$ is the unit disc of radius one with centre at the origin that

$$\iint_{\mathcal{R}} xy \, dA = 0.$$

[6 Marks]

2. (a) Corresponding values of x and y for a polynomial function are given in a table on page seven of this examination paper. There is an error in the table. Form a *forward difference* table up as far as and including second differences for these values. Include the completed table with your answer sheet.

   i. Locate and correct the error in the table.

   [5 marks]

   ii. Extend the table to estimate the values of y at $x = 0$ and $x = 120$.

   [2 marks]

   iii. Use *linear interpolation* to estimate the value of y at $x = 68$.

   [3 marks]

   iv. Use the *Newton-Gregory Interpolation formula* to approximate the value of y at $x = 84$.

   [3 marks]

   v. Estimate the value of $y'(x)$ at $x = 96$.

   [2 marks]

   (b) World concrete production has been increasing over the five decades from 1971 to 2011:

decades after 1971, t	0	1	2	3	4
concrete production in Gt, C	3.8	5.9	7	10.7	18.8

   It is believed that t and C have a relationship of the form:

$$C = k\,R^t.$$

   i. Find the best values of the constants k and R in the *least squares sense*. Use two places of decimals for all calculations.

   [8 marks]

   ii. Hence make a prediction for world concrete production in 2021.

   [2 Marks]

3. (a) Use *Gaussian elimination* to find the solution of the simultaneous equations

$$\begin{aligned} A + 2B + C + 2D &= 7 \\ 3A + 8B + 4C + 4D &= 11 \\ 2A + 6B + 5C + 6D &= 16 \\ 4A + 6B + 4C + 7D &= 24 \end{aligned}$$

[8 marks]

(b) Consider the linear system

$$\begin{aligned} 3.4x + 0.8y + 1.2z &= 1.3 \\ 1.2x - 4.2y + 0.0z &= 2.0 \\ 6.2x + 0.4y - 9.0z &= -1.0 \end{aligned}$$

i. Is the linear system *diagonally dominant*? Justify your answer. [1 Mark]

ii. Use the associated diagonal system to give, to two decimal places, an approximate solution (x_0, y_0, z_0) to the linear system. [2 Marks]

iii. Use this initial approximation and

two iterations of *Jacobi's Method*

OR

two iterations of the *Gauss-Siedel Method*

to estimate the solution of the linear system.

If you were unable to do part ii. use $(x_0, y_0, z_0) = (0.4, -0.5, 0.1)$.
Use two places of decimals for all calculations. [6 marks]

iv. To find a more accurate and indeed exact solution Gaussian elimination is required. However Gaussian elimination is very sensitive to rounding error. To counteract these problems, *Gaussian Elimination with Partial Pivoting* can be used.

Use Gaussian Elimination with Partial Pivoting, and two places of decimals, to find the solution set of the linear system:

$$\begin{pmatrix} 3.4 & 0.8 & 1.2 \\ 1.2 & -4.2 & 0.0 \\ 6.2 & 0.4 & -9.0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1.3 \\ 2.0 \\ -1.0 \end{pmatrix}.$$

Use two places of decimals for all calculations. [8 marks]

4. Use *Laplace Methods* to solve the following differential equations

   (a) $\dfrac{dy}{dx} + y(x) = 6, \quad y(0) = 4.$

   [7 marks]

   (b) $x''(t) + x'(t) - 6x(t) = 0, \quad x(0) = 0,\ x'(0) = -2.$

   [9 marks]

   (c) $\dfrac{d^2y}{dt^2} + 8\dfrac{dy}{dt} + 25y(t) = 0, \quad y(0) = 0,\ y'(0) = 1.$

   [9 marks]

5. (a) Evaluate the *line integral*

$$\int_{\mathcal{C}} 6x^2\,dx + 12xy\,dy$$

   where $\mathcal{C}$ is

   i. the line segment passing from $(0,0)$ to $(2,1)$.

   [3 marks]

   ii. the arc of $y = x^2$ passing from $(0,0)$ to $(3,9)$.

   [4 marks]

   iii. the arc of the circle $x^2 + y^2 = 1$ passing from $(1,0)$ to $(0,1)$.

   [4 marks]

   (b) Sketch the triangular region with vertices $(0,0)$, $(0,2)$ and $(2,0)$.

   By evaluating an appropriate double integral, find the *second moment of area* of this region about the x-axis.

   [HINT: $I_{xx} = \displaystyle\int y^2\,dA$]

   [7 marks]

   (c) A cylinder is described by

$$x^2 + y^2 \leq 4 \quad \text{and} \quad 0 \leq z \leq 3.$$

   For this volume, evaluate the *triple integral*

$$\iiint_V y^2 z\,dV.$$

   [7 marks]

$$\left[\text{HINT: } \cos^2 A = \frac{1}{2}(1 + \cos 2A) \text{ and } \sin^2 A = \frac{1}{2}(1 - \cos 2A)\right]$$