MATH7019: Sample Test 1

Name:
Student Number:

Answer all questions. Marks may be lost if necessary work is not clearly shown. There are a set of tables located at the back of this sample test.

1. Find the first three terms of the Taylor Series of $f(x) = \ln(\sec x + \tan x)$ about the point $x = \dfrac{\pi}{4}$.

   $\Big[$NOTE: $f(x) = f(a) + f'(a)(x-a) + \dfrac{f''(a)}{2!}(x-a)^2 + \cdots$;
   $(\sec x)' = \sec x \tan x$; $\sec x = 1/\cos x$; $\sin(\pi/4) = \cos(\pi/4) = 1/\sqrt{2}$; $\tan(\pi/4) = 1\Big]$

   [5 Marks]

   Solution:

2. If square beams of a given width and alloy are of length L metres, then the maximum deflection under a uniform loading of W kN m^{-1} in millimetres is given by

$$\delta_{\max} = \frac{WL^4}{25920}.$$

Estimate a range of values for $\delta_{\max}$ where the loading was measured to be 40 kN m^{-1} and the length was measured to be 6 m with maximum errors of 0.1 kN m^{-1} and 0.01 m.

[6 Marks]

Solution:

3. Find the Taylor Series Expansion of $f(x,y) = x\ln(x-2y)$ about $(3,1)$.

$$\left[\text{NOTE: } f(x,y) \approx f(a,b) + (x-a)f_x + (y-b)f_y + \frac{(x-a)^2}{2!}f_{xx} + (x-a)(y-b)f_{xy} + \frac{(y-b)^2}{2!}f_{yy}\right]$$

[8 Marks]

Solution:

4. Using a step-size of $h = 0.1$, numerically approximate $y(0.2)$ where $y(x)$ is the solution of the initial value problem

$$\frac{dy}{dx} = x^2 + y(x)^2 \text{ , and } y(0) = 1.$$

$$\left[\text{NOTE: } y_{k+1} = y_k + y_k' h + \frac{y_k''}{2} h^2.\right]$$

[6 Marks]

Solution:

5. Find the general solution of the homogenous second order ordinary linear differential equations:

(a) $2\dfrac{d^2y}{dx^2} + 6\dfrac{dy}{dx} + 4y(x) = 0$

(b) $\dfrac{d^2\theta}{dt^2} + 9\theta(t) = 0$

[4 Marks each]

Solution:

Roughwork