CORK INSTITUTE OF TECHNOLOGY
INSTITIÚID TEICNEOLAÍOCHTA CHORCAÍ

Semester 1 Examinations 2012/13

Module Title: Technological Mathematics 312

Module Code: MATH 7021

School: Building & Civil Engineering

Programme Title: Bachelor of Engineering in Civil Engineering – Year 3

Programme Code: CCIVL_7_Y3

External Examiner(s): Mr. C. O'Sullivan

Internal Examiner(s): Mr. J.P. Mc Carthy

Instructions: Answer four questions. All questions carry equal marks

Duration: 2 Hours

Sitting: Winter 2012

Exam Requirements: Mathematics Tables

Note to Candidates: Please check the Programme Title and the Module Title to ensure that you are attempting the correct examination.
If in doubt please contact an Invigilator.

1. (a) It is believed that variables y and x have a linear relationship of the form

$$y = mx + c$$

for constants m and c. For the data below find the line of best fit *in the least squares sense* and hence write down the values of m and c.

x	1	2	3	4	5	6
y	7	10	12	15	19	21

(7 marks)

(b) Consider the following linear system:

$$\begin{aligned} 0.01x + 0.80y &= -0.94 \\ 1.20x + 0.04y &= +2.80 \end{aligned}$$

Suppose that we can only make calculations correct to two decimal places. What would then be an appropriate method to find the solution correct to one decimal place?

(2 marks)

Implement this method to estimate the values of x and y correct to **one decimal place.**

(5 marks)

Use *Cramer's Rule* to find the correct value of x and compare this with your estimate for x.

(2 marks)

(c) Suppose that the charge $q(t)$ on a capacitor satisfies the first order differential equation:

$$\frac{dq}{dt} + 2q(t) = 2.$$

Use *Laplace Methods* to solve the differential equation for the charge on the capacitor plate $q(t)$ at any time t if the plate is initially uncharged $q(0) = 0$.

(9 marks)

2. (a) Corresponding values of x and y for a polynomial function are given in a table attached to this examination paper. There is an error in the table. Form a *forward difference* table up as far as and including second differences for these values. Include the completed table with your answer sheet.

   i. Locate and correct the error in the table.

   (5 marks)

   ii. Extend the table to estimate the values of y at $x = 0$ and $x = 6$.

   (2 marks)

   iii. Use *linear interpolation* to estimate the value of y at $x = 3.4$.

   (3 marks)

   iv. Use the *Newton-Gregory Interpolation formula* to approximate the value of y at $x = 4.2$.

   (3 marks)

   v. Estimate the value of $y'(x)$ at $x = 4.8$.

   (2 marks)

   (b) The following inputs X and outputs Y were measured and recorded:

input, X	-1	0	2
output, Y	14	0	31

   It is believed that X and Y have a relationship of the form:

$$Y = AX^2 + BX.$$

   i. Find the best values of A and B in the *least squares sense.*

   (5 marks)

   ii. Hence estimate the value of Y at $X = 1$.

   (1 mark)

   iii. Also, use *Lagrangian Interpolation* to estimate the value of Y at $X = 1$.

   (3 marks)

   iv. Is the Lagrange Interpolation estimate equal to the Least Squares estimate? Explain why.

   (1 mark)

3. (a) Use *Gaussian Elimination* to find the solutions of the simultaneous equations

$$\begin{aligned} x + y + z + w &= 2 \\ 2x - 2z + 4w &= 14 \\ x + 2z - 3w &= -4 \\ 2x + y - 3z + 4w &= 16 \end{aligned}$$

[HINT: Do **not** use decimal rounding]

(8 marks)

(b) Use *Gaussian Elimination with Partial Pivoting* to estimate, **correct to two decimal places**, the solution set of the linear system:

$$\begin{pmatrix} 2 & 1 & 4 \\ 5 & 2 & 6 \\ 6 & 6 & 9 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}.$$

(7 marks)

(c) Consider the set of simultaneous equations:

$$\begin{aligned} 10x - 2y + 3z &= 28 \\ 3x + 15y - z &= -4 \\ x - y + 6z &= 21 \end{aligned}$$

Explain briefly why we could use $(x_0, y_0, z_0) = (2.8, -0.27, 3.5)$ as a first approximation to the solution.

(1 mark)

Use **three iterations** of *Jacobi's Method* **or two iterations** of the *Gauss-Siedel Method* to estimate **correct to two decimal places** the solution of simultaneous equations.

(6 marks)

(d) Suppose that a is a non-zero constant and

$$\begin{aligned} ax + 2y &= 2 \\ -3ax + y &= 3 \end{aligned}$$

Using *Cramer's Rule*, or otherwise, show that $x = -\dfrac{4}{7a}$.

(3 marks)

4. Use *Laplace Methods* to solve the following differential equations

   (a) $\dfrac{dx}{dt} - x(t) = 2e^{3t} \qquad x(0) = 1.$

   (7 marks)

   (b) $\dfrac{d^2x}{dt^2} + 2\dfrac{dx}{dt} + 10x(t) = 0 \qquad x(0) = 0,\, x'(0) = 6.$

   (9 marks)

   (c) $\dfrac{d^2y}{dx^2} + 3\dfrac{dy}{dx} + 2y(x) = 80\cos 2x \qquad y(0) = y'(0) = 0.$

   (9 marks)

5. (a) Sketch the triangular region with vertices $(-1, 0)$, $(1, 0)$ and $(0, 4)$.

   (1 mark)

   i. Evaluate the *line integral*

   $$\oint_C (6x^2\,dx + 12y^2\,dy)$$

   where C is the perimeter of this region.

   (6 marks)

   ii. By evaluating an appropriate double integral, find the *second moment of area* of this region about the x-axis.

   (6 marks)

   (b) A cylinder is described by

   $$x^2 + y^2 \leq 9 \quad \text{and} \quad 0 \leq z \leq 2.$$

   i. Evaluate the *line integral*

   $$\oint_C (12xy\,dx + 24y^2\,dy),$$

   where C is the perimeter of the base of the cylinder.

   (6 marks)

   ii. For this volume, evaluate the *triple integral*

   $$\iiint\limits_V x^2 z\,dV.$$

   (6 marks)

$$\left[\text{HINT: } \cos^2 A = \frac{1}{2}(1 + \cos 2A) \text{ and } \sin^2 A = \frac{1}{2}(1 - \cos 2A)\right]$$

LAPLACE TRANSFORMS-*

For a function f(t) the Laplace Transform of f(t) is a function in s defined by

$$F(s) = \int_0^\infty e^{-st} f(t)dt \quad \text{where } s>0.$$

f(t)	F(s)
A=constant	$\frac{A}{s}$
t^N	$\frac{N!}{s^{N+1}}$
e^{at}	$\frac{1}{s-a}$
sinhkt	$\frac{k}{s^2-k^2}$
coshkt	$\frac{s}{s^2-k^2}$
$\sin\omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos\omega t$	$\frac{s}{s^2+\omega^2}$
$e^{at}f(t)$	F(s-a)
$f'(t)$	sF(s)-f(0)
$f''(t)$	$s^2F(s)-sf(0)-f'(o)$

Note: $\cosh A = \frac{e^A + e^{-A}}{2}$ $\qquad \sinh A = \frac{e^A - e^{-A}}{2}$

DERIVATIVES

f(x) a=constant	f′(x)
x^n	nx^{n-1}
lnx	$\frac{1}{x}$
e^{ax}	$a e^{ax}$
sinx	cosx
cosx	-sinx

INTEGRALS

f(x) a=constant	$\int f(x)dx$
x^n	$\frac{x^{n+1}}{n+1}$ if $n \neq -1$
$\frac{1}{x}$	lnx
e^{ax}	$\frac{1}{a} a e^{ax}$
sinx	-cosx
cosx	sinx

INTERPOLATION

$$f(x) = \frac{(x - x_1)(x - x_2)}{(x_0 - x_1)(x_0 - x_2)} f(x_0) + \frac{(x - x_0)(x - x_2)}{(x_1 - x_0)(x_1 - x_2)} f(x_1) + \frac{(x - x_0)(x - x_1)}{(x_2 - x_0)(x_2 - x_1)} f(x_2)$$

$$f(x_0 + rh) = f(x_0) + r\Delta f_0 + \frac{r(r-1)}{2!}\Delta^2 f_0 + ...$$

$$f'(x_0 + rh) = \frac{1}{h}\left[\Delta f_0 + \frac{(2r-1)}{2!}\Delta^2 f_0 + ...\right]$$

Name: ______________________

x	y	Δy	$\Delta^2 y$
0.0			
0.5	-9		
1.0	-8		
1.5	-3		
2.0	6		
2.5	19		
3.0	34		
3.5	57		
4.0	82		
4.5	111		
5.0	144		
5.5	181		
6.0			