CORK INSTITUTE OF TECHNOLOGY
INSTITIÚID TEICNEOLAÍOCHTA CHORCAÍ

Semester 1 Examinations 2011/12

Module Title: Mathematics for Science 2.1

Module Code: **MATH 6037**

School: Science & Informatics

Programme Title:
Bachelor of Science (Honours) in Environmental Science and Sustainable Technology- Year 2
Bachelor of Science (Honours) in Instrument Engineering- Year 2
Bachelor of Science in Applied Physics & Instrumentation – Year 2
Higher Certificate in Industrial Measurement & Control – Year 2

Programme Code:
SESST_8_Y2
SINEN_8_Y2
SPHYS_7_Y2
SIMCT_6_Y2

External Examiner(s): **Dr. P. Robinson**

Internal Examiner(s): **Dr. M. Brennan**

Instructions: Answer Q1 (COMPUSORY) and any other 2 questions.
Q1 carries 50 marks. All other questions carry 25 marks.

Duration: 2 Hours

Sitting: Winter 2011

Requirements for this examination: Mathematical Tables

Note to Candidates: Please check the Programme Title and the Module Title to ensure that you have received the correct examination.
If in doubt please contact an Invigilator

Q1. (a) Use *partial fractions* to decompose

$$\frac{7x^2+5x-6}{x^2(x-3)}$$

(9 marks)

(b) Use *the cover up method* to find

$$\mathcal{L}^{-1}\left\{\frac{s^2+2s+3}{s(s-1)(s+2)}\right\}$$

(7 marks)

(c) Find the inverse Laplace transforms

(i) $\mathcal{L}^{-1}\left\{\frac{(s+3)^2}{s^4}\right\}$ (ii) $\mathcal{L}^{-1}\left\{\frac{s+2}{s^2+36}\right\}$ (iii) $\mathcal{L}^{-1}\left\{\frac{s}{(s-2)^3}\right\}$

(14 marks)

(d) Solve using **only** the *Laplace Transform Method* for $x(t)$,

$$\frac{dx}{dt}+5x=40e^{-5t}, \qquad x(0)=0.$$

(11 marks)

(e) A Local Authority allows up to 18 houses per hectare (1 hectare=10000m^2). From one straight boundary of a plot of land the following offsets were taken at 50m intervals.

Distance (m)	0	50	100	150	200	520	300	350	400
Offset (m)	56	62	72	85	96	100	93	88	70

(i) Use Simpson's Rule to estimate the area of the site.

(ii) Calculate the maximum number of houses that may be built.

(9 marks)

Q2. (a) Use *integration by parts* to determine $\int_0^{\frac{\pi}{4}} x\cos 4x\,dx$.

(7 marks)

(b) Use the *definition* of Laplace transform to derive $\mathcal{L}\{e^{3t}\}$.

(6 marks)

(c) Find the Laplace transform of the following functions.

(i) $(2t-3)(t+2)$ (ii) $4t^3e^{2t}$ (iii) $\sin(4t)\cos(2t)$

(12 marks)

Q3. (a) The differential equation governing the displacement $x(t)$ of a damped oscillator is given by

$$x''(t) + x'(t) + \frac{17}{4}x(t) = 0.$$

(i) Solve the differential equation using the *Laplace Transform Method*, given that $x(0) = 0$ and $x'(0) = 3$.

(ii) Determine the period of the oscillation and the duration of the oscillations.

(iii) Sketch $x(t)$ labelling the axes appropriately.

(16 marks)

(b) The modulus of rigidity,

$$G = \frac{R^4\theta}{L}$$

where R is the radius, θ is the angle of twist and L is the length. Use partial differentials to determine an approximate expression for ΔG. Hence find the *approximate* percentage change in G when R is increased by 2%, θ is reduced by 5% an dL is increased by 4%.

(9 marks)

Q4. (a) Determine the partial derivatives.

(i) $\dfrac{\partial}{\partial x}(5x^3 e^{2x^2 y})$

(ii) $\dfrac{\partial}{\partial y}\left(\dfrac{1}{\sqrt{x^2 - y^2}}\right)$

(8 marks)

(b) Use *Euler's method* with step size $h = 0.2$ to estimate $y(0.5)$, where $y(x)$ is the solution to the initial-value problem,

$$(1 + x^3)\frac{dy}{dx} - 2y = 0$$
$$y(0.1) = 1.$$

(9 marks)

(c) Verify that the equation

$$3x^3 - 10x - 14 = 0$$

has a root in the interval [2,3]. Use the *Newton-Raphson method* to approximate this root correct to *three* decimal places where $x_0 = 2$.

(8 marks)

Laplace Transform Formulae

$$\mathcal{L}\{f(t)\} \equiv F(s) = \int_0^\infty f(t)e^{-st}\,dt = \int_0^\infty e^{-st}f(t)\,dt \qquad \textit{Definition}$$

$$\mathcal{L}\{Af(t) + Bg(t)\} = AF(s) + BG(s), \quad A, B \text{ are constants} \qquad \textit{Linearity}$$

$$\mathcal{L}\{\dot{f}(t)\} = sF(s) - f(0)$$

$$\mathcal{L}\{\ddot{f}(t)\} = s^2F(s) - sf(0) - \dot{f}(0)$$

$$\mathcal{L}\{f(t)e^{at}\} = \mathcal{L}\{f(t)\}|_{s\to s-a} = F(s)|_{s\to s-a} \qquad \textit{First Translation Theorem}$$

$$\mathcal{L}\{f(t-a)\mathcal{U}(t-a)\} = e^{-as}\mathcal{L}\{f(t)\} = e^{-as}F(s) \qquad \textit{Second Translation Theorem}$$

$$\mathcal{L}\{1\} = \frac{1}{s}$$

$$\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}, \qquad n = 0, 1, 2, 3, \ldots$$

$$\mathcal{L}\{e^{at}\} = \frac{1}{s-a} \qquad a \text{ is a constant}$$

$$\mathcal{L}\left\{\int_0^t f(w)\,dw\right\} = \frac{F(s)}{s}$$

$$\mathcal{L}\{\delta(t-a)\} = e^{-sa}$$

$$\mathcal{L}\{\mathcal{U}(t-a)\} = \frac{e^{-sa}}{s} \qquad a > 0,\ a \text{ is a positive constant}$$

$$\mathcal{L}\{\sin kt\} = \frac{k}{s^2+k^2} \qquad k \text{ is a constant}$$

$$\mathcal{L}\{\cos kt\} = \frac{s}{s^2+k^2}$$

$$\mathcal{L}\{\sinh kt\} = \frac{k}{s^2-k^2}$$

$$\mathcal{L}\{\cosh kt\} = \frac{s}{s^2-k^2}$$