CORK INSTITUTE OF TECHNOLOGY
INSTITIÚID TEICNEOLAÍOCHTA CHORCAÍ

Autumn Examinations 2011/12

Module Title: Mathematics for Science 2.1

Module Code: **MATH 6037**

School: Science and Informatics

Programme Title:
Bachelor of Science (Honours) in Environmental Science and Sustainable Technology – Year 2
Bachelor of Science (Honours) in Instrument Engineering – Year 2
Bachelor of Science in Applied Physics and Instrumentation – Year 2
Higher Certificate in Industrial Measurement & Control – Year 2

Programme Code:
SESST_8_Y2
SINEN_8_Y2
SPHYS_7_Y2
SIMCT_6_Y2

External Examiner(s): Dr. P. Robinson

Internal Examiner(s): Dr. M. Brennan

Instructions: Answer Q1 (compulsory) and any 2 other questions.
Q1 is worth 50 marks. All other questions are worth 25 marks each.

Duration: **2 HOURS**

Sitting: Autumn 2012

Requirements for this examination:

Note to Candidates: Please check the Programme Title and the Module Title to ensure that you have received the correct examination paper.
If in doubt please contact an Invigilator.

Q1. (a) Use *partial fractions* to decompose

$$\frac{6x^2 - x + 64}{x(x^2 + 16)}$$

(9 marks)

(b) Use *complete the square* to find

$$\mathcal{L}^{-1}\left\{\frac{2s}{s^2 + 6s + 25}\right\}$$

(8 marks)

(c) Find the inverse Laplace transforms

(i) $\mathcal{L}^{-1}\left\{\frac{(s+1)^2}{s^3}\right\}$ (ii) $\mathcal{L}^{-1}\left\{\frac{3s+2}{s^2+16}\right\}$ (iii) $\mathcal{L}^{-1}\left\{\frac{s}{(s+3)^2}\right\}$

(14 marks)

(d) Solve using **only** the *Laplace Transform Method* for $y(t)$,

$$\frac{dy}{dt} + 4y = 32e^{-4t}, \qquad y(0) = 0.$$

(11 marks)

(e) Use the Midpoint Rule with $n = 6$, to approximate the integral

$$\int_0^3 e^{-x^2}\, dx.$$

(8 marks)

Q2. (a) Use *integration by parts* to determine $\int_0^2 xe^{2x}\,dx$.

(7 marks)

(b) Use the *definition* of Laplace transform to derive $\mathcal{L}\{5t\}$.

(8 marks)

(c) Find the Laplace transform of the following functions.

(i) $(t+1)^2 e^{3t}$ (ii) $\cos(5t)\cos(3t)$

(10 marks)

Q3. (a) The differential equation governing the displacement $x(t)$ of a damped oscillator is given by

$$x''(t) + 4x'(t) + 13x(t) = 0.$$

(i) Solve the differential equation using the *Laplace Transform Method*, given that $x(0) = 0$ and $x'(0) = 4$.

(ii) Determine the period of the oscillation and the duration of the oscillations.

(iii) Sketch $x(t)$ labelling the axes appropriately.

(16 marks)

(b) A rectangular box has sides of length $x, y, z\,mm$. Sides x and y are *expanding* at rates of 1.5 and 2.5 mm/s respectively, while side z is *contracting* at a rate of 0.25 mm/s. Use partial derivatives to determine the rate of change of volume $\frac{dV}{dt}\,mm^3/s$ when $x = 15\,mm, y = 12\,mm$ and $z = 10\,mm$.

(9 marks)

Q4. (a) Determine the partial derivatives.

(i) $\dfrac{\partial}{\partial x}\sin^2(x^3y)$

(ii) $\dfrac{\partial}{\partial y}\left(y\sqrt{x^2-y^2}\right)$

(9 marks)

(b) Use *Euler's method* with step size $h = 0.3$ to estimate $y(0.9)$, where $y(x)$ is the solution to the initial-value problem,

$$(1+x^2)\frac{dy}{dx} - 3y = 0$$
$$y(0) = 1.$$

(8 marks)

(c) Verify that the equation

$$2x^3 + 5x - 11 = 0$$

has a root in the interval [1,2]. Use the *Newton-Raphson method* to approximate this root correct to *four* decimal places where $x_0 = 1.0$.

(8 marks)

Laplace Transform Formulae

$$\mathcal{L}\{f(t)\} \equiv F(s) = \int_0^\infty f(t)e^{-st}\,dt = \int_0^\infty e^{-st}f(t)\,dt \qquad \textit{Definition}$$

$$\mathcal{L}\{Af(t) + Bg(t)\} = AF(s) + BG(s), \quad A, B \text{ are constants} \qquad \textit{Linearity}$$

$$\mathcal{L}\{\dot{f}(t)\} = sF(s) - f(0)$$

$$\mathcal{L}\{\ddot{f}(t)\} = s^2F(s) - sf(0) - \dot{f}(0)$$

$$\mathcal{L}\{f(t)e^{at}\} = \mathcal{L}\{f(t)\}|_{s\to s-a} = F(s)|_{s\to s-a} \qquad \textit{First Translation Theorem}$$

$$\mathcal{L}\{f(t-a)\mathcal{U}(t-a)\} = e^{-as}\mathcal{L}\{f(t)\} = e^{-as}F(s) \qquad \textit{Second Translation Theorem}$$

$$\mathcal{L}\{1\} = \frac{1}{s}$$

$$\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}, \qquad n = 0, 1, 2, 3, \ldots$$

$$\mathcal{L}\{e^{at}\} = \frac{1}{s-a} \qquad a \text{ is a constant}$$

$$\mathcal{L}\left\{\int_0^t f(w)\,dw\right\} = \frac{F(s)}{s}$$

$$\mathcal{L}\{\delta(t-a)\} = e^{-sa}$$

$$\mathcal{L}\{\mathcal{U}(t-a)\} = \frac{e^{-sa}}{s} \qquad a > 0,\ a \text{ is a positive constant}$$

$$\mathcal{L}\{\sin kt\} = \frac{k}{s^2+k^2} \qquad k \text{ is a constant}$$

$$\mathcal{L}\{\cos kt\} = \frac{s}{s^2+k^2}$$

$$\mathcal{L}\{\sinh kt\} = \frac{k}{s^2-k^2}$$

$$\mathcal{L}\{\cosh kt\} = \frac{s}{s^2-k^2}$$