CORK INSTITUTE OF TECHNOLOGY

INSTITIÚID TEICNEOLAÍOCHTA CHORCAÍ

Autumn Examinations 2011/2012

Module Title: Technological Mathematics 201

Module Code:	**MATH6040**
School:	School of Mechanical & Process Engineering School of Manufacturing, Biomedical and Facilities Engineering School of Civil, Structural and Environmental Engineering
Programme Title:	Bachelor of Engineering in Mechanical Engineering Bachelor of Engineering in Building Services Engineering Bachelor of Engineering in Biomedical Engineering Bachelor of Engineering in Sustainable Energy Technology Bachelor of Engineering in Civil Engineering C&G Tech Awards Advanced Diploma
Programme Code:	EMECH_7_Y2 EBSEN_7_Y2 EBIME_7_Y2 ESENT_8_Y2 CCIVL_7_Y2 EAMAT_6_YX
External Examiner(s):	**Dr. Padraig Kirwan**
Internal Examiner(s):	**Ms. J. English, Dr. M. Lishchynska, Dr. J. O'Donovan**
Instructions:	**Answer Question 1 (compulsory) (worth 40 marks) AND one question from section B (worth 30 marks) AND one question from section C (worth 30 marks).**

Duration: 2 hours

Sitting: Autumn 2012

Requirements for this examination: Mathematical Tables.

Note to Candidates: Please check the **Programme Title** and the **Module Title** to ensure that you have received the correct examination.

If in doubt please contact an Invigilator.

Section A

Q1.

(a) Differentiate the following function implicitely

$$x^3 + y^3 = 3xy\,.$$

[6 marks]

(b) A curve is described parametrically by the equations

$$x = 3t^2; \quad y = 3t\,.$$

Find the slope of the tangent line to this curve at the point (24, 6)

[5 marks]

(c) Find the following integral

$$\int 3xe^{2x}\,dx$$

[6 marks]

(d) Evaluate the following integral

$$\int \frac{1}{x^2 + 6x + 13}\,dx$$

[5 marks]

(e) For $\mathbf{x} = 3\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$ and $\mathbf{y} = 2\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$, find

(i) $\mathbf{x} + \mathbf{y}$

(ii) $\mathbf{x} - \mathbf{y}$

(iii) $(\mathbf{x} + \mathbf{y}) \bullet (\mathbf{x} - \mathbf{y})$

[4 marks]

(f) For the vectors $\mathbf{a} = 3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ and $\mathbf{b} = 2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$, find $\mathbf{a} \times \mathbf{b}$.

[4 marks]

P.T.O.

(g) Express the system of equations

$$2x + y = 3$$
$$x - 2y = -1$$

in matrix form. Use the inverse matrix method to solve it and check your answer.

[5 marks]

(h) For the matrices

$$B = \begin{pmatrix} 2 & 1 & -3 \\ 3 & -2 & 4 \end{pmatrix}; \quad C = \begin{pmatrix} 1 & 0 & 3 \\ -2 & 1 & 2 \\ 4 & 5 & -3 \end{pmatrix}$$

show that $(BC)^T = C^T B^T$

[5 marks]

Section B

Q2.

(a) For the matrices

$$A = \begin{pmatrix} 2 & 1 & 0 \\ 3 & -2 & -1 \end{pmatrix}; B = \begin{pmatrix} 1 & 2 & 1 \\ -3 & 2 & 0 \end{pmatrix}; \quad C = \begin{pmatrix} 5 & 3 \\ 2 & -1 \\ 3 & 4 \end{pmatrix}$$

(i) Determine each of the following, if defined:

- $A + B$
- $A + C$
- $(A + B)^T$
- CA
- AC

(ii) Given the matrices

$$A = \begin{pmatrix} 1 & -3 \\ 4 & 6 \end{pmatrix}; \quad C = \begin{pmatrix} 16 & 32 \\ 36 & 84 \end{pmatrix}$$

Determine the matrix X such that $XA = C$

[11 marks]

(b) Find the values of k for which the determinant of the matrix

$$\begin{bmatrix} 1 & (1+k) \\ 2k & 4 \end{bmatrix}$$

is equal to zero.

[3 marks]

(c) (i) Obtain the inverse of the matrix

$$A = \begin{pmatrix} 3 & 2 & -2 \\ 4 & 3 & 3 \\ -2 & 1 & -1 \end{pmatrix}$$

P.T.O.

Use this to solve the system of equations

$$3x + 2y - 2z = 16$$
$$4x + 3y + 3z = 2$$
$$-2x + y - z = 1$$

(ii) Use Cramer's rule to verify the answer for z.

[16 marks]

Q3.

(a) For the vector $\mathbf{v} = 3\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}$:

(i) Find the magnitude of $\mathbf{v}$.

(ii) Find the unit vector in the direction of $\mathbf{v}$.

(iii) If the vector $\mathbf{w} = 4\mathbf{i} - 3\mathbf{j} + t\mathbf{k}$ is perpendicular to $\mathbf{v}$, find the value of t.

[10 marks]

(b) Find the work done by a force $\mathbf{F} = \mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$ in moving an object from the point A(1, 1, 1) to the point B(2, 2, 2).
(You may assume that force is measured in Newtons and displacement is measured in metres)

[10 marks]

(c) A force of 18 Newtons acts through the point P(-1, 3, -4) in the direction of the vector $3\mathbf{i} + 6\mathbf{j} - 6\mathbf{k}$. Find its moment about the origin (0, 0, 0).
(You may assume that displacement is measured in metres)

[10 marks]

Section C

Q4.

a) Given $z = x^3 + xy^3$ find $\frac{\partial^2 z}{\partial x^2}$, $\frac{\partial^2 z}{\partial y^2}$, $\frac{\partial^2 z}{\partial y\,\partial x}$ and $\frac{\partial^2 z}{\partial x\,\partial y}$.

[12 marks]

b) The surface area A of a spherical balloon is increasing at the rate of 40 cm^2/s. Find the rate of change of the radius R of the balloon when the radius equals 20 cm.

[10 marks]

c) Given the function $z = \ln(e^x + e^y)$ show that

$$\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 1$$

[8 marks]

Q5.

a) Find the following integral

$$\int \ln x \, dx$$

[6 marks]

b) The force F (Newtons) acting on an object is given by $F(x) = \dfrac{12}{4x^2 + 9}$ where x is the displacement of the object in metres. Find the work done in moving the object by 3 metres from the origin O.

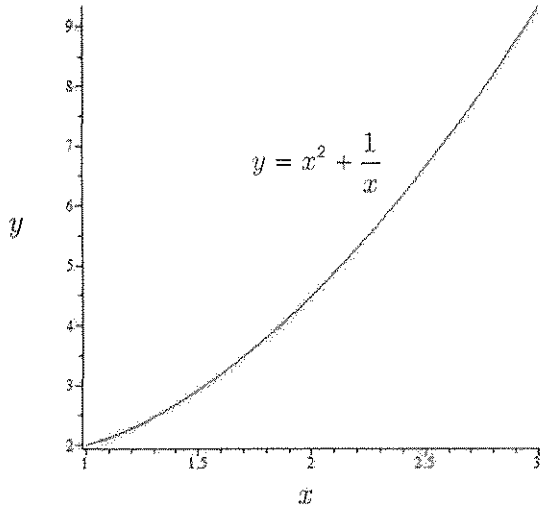
[7 marks]

c) Given the curve $y = x^2 + \dfrac{1}{x}$ (shown on the picture) determine

(i) the area between this curve, the x-axis, and the ordinates at $x = 1$ and $x = 3$.

(ii) the coordinates of the centroid of this area.

[17 marks]



Required Formulae

Centroid: $\overline{x} = \dfrac{\int xy\,dx}{\int y\,dx}$; $\overline{y} = \dfrac{\frac{1}{2}\int y^2 dx}{\int y\,dx}$

Work done by variable force: $W = \int F dx$