MATH7021: Sample Test 1

Name:
Student Number:

Answer all questions. All questions worth equal marks. Marks may be lost if necessary work is not clearly shown.

$$f(x_0 + rh) = f(x_0) + r\,\Delta f_0 + \frac{r(r-1)}{2!}\Delta^2 f_0.$$

1. The following inputs x and outputs y of a polynomial function were measured and recorded:

x	y	Δy	Δ^2y
0			
10	1		
20	3		
30	7		
40	13		
50			

Form a *forward difference* table up as far as and including second differences for these values.
[2 Marks]

(a) Extend the table to estimate the values of y at $x = 0$ and $x = 50$.
[1 mark]

(b) Use *linear interpolation* to estimate the value of y at $x = 37$.
[2 marks]

(c) Use the *Newton-Gregory Interpolation formula* to approximate the value of y at $x = 21$.
[3 marks]

Solution:

2. The following inputs L and outputs N were measured and recorded:

input, L	1	2	3	4	5
output, N	-2.00	-0.25	-0.07	-0.03	-0.01

It is believed that L and N are related by a law of the form $N = aL^b$ for constants a and b. Find the best values of a and b in the Least Squares sense using two or more places of decimals.

Solution:

3. Use *Gaussian Elimination* to find the solutions of the simultaneous equations

$$\begin{aligned} x + y + z + w &= 2 \\ 2x - 2z + 4w &= 14 \\ x + 2z - 3w &= -4 \\ 2x + y - 3z + 4w &= 16 \end{aligned}$$

Solution:

4. Use *Gaussian Elimination with Partial Pivoting* to estimate, **correct to two decimal places**, the solution set of the linear system:

$$\begin{pmatrix} 2 & 1 & 4 \\ 5 & 2 & 6 \\ 6 & 6 & 9 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}.$$

Solution: