MATH7021: Sample Test 1

Name:
Student Number:

Answer all questions. All questions worth equal marks. Marks may be lost if necessary work is not clearly shown. There are a set of Finite Difference Tables as well as Laplace Transform Tables located at the back of this sample test.

1. Corresponding values of x and y *for a polynomial function* are given in a table attached to this examination paper. There is an error in the table. Form a forward difference table up as far as and including second differences for these values. Include the completed table with your answer sheet.

   (a) Locate and correct the error in the table.

   (b) Extend the table to calculate the values of y at $x = 0$ and $x = 6$.

   (c) Use linear interpolation to estimate the value of y at $x = 3.4$. Also use the Newton-Gregory Interpolation formula to approximate the value of y at $x = 3.4$.

   Interpolation Formula: $f(x_0 + h) \approx f(x_0) + r\Delta f_0 + \dfrac{r(r-1)}{2}\Delta^2 f_0$

   Solution:

2. Use Gaussian Elimination *with partial pivoting* to solve the set of simultaneous equations below. All calculations should be correct to two decimal places.

$$2x + y + 3z = 9$$
$$2x + 3y + 6z = 13$$
$$5x + 10y + 6z = 12$$

Solution:

3. By approximating to a diagonal linear system, find, correct to two decimal places, an approximate solution (x_0, y_0, z_0) to the linear system

$$\begin{pmatrix} 13 & 1 & -4 \\ 3 & 11 & 2 \\ 1 & 6 & 14 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -3 \\ -12 \\ 38 \end{pmatrix}$$

Use three iterations of Jacobi's Method *or* three iterations of the Gauss-Siedel Method to estimate correct to two decimal places the solution of the linear system.

Solution:

4. Find the Laplace transforms of the functions which satisfy the following differential equations:

$$(i)\ \ 4\frac{dI}{dt} + 12I = 60\,,\ I(0) = 0.$$

$$(ii)\ \ y'' + 2y' + 4y = 0\,,\ y(0) = 1,\ y'(0) = 0.$$

Solution: