**CORK INSTITUTE OF TECHNOLOGY**
**INSTITIÚID TEICNEOLAÍOCHTA CHORCAÍ**

**Semester 2 Examinations 2008/9**

# Module Title: Mathematics for Science 2.2 with Maple

**Module Code:** **MATH 6038**

**School:** School of Science

**Programme Title:**
Bachelor of Science (Honours) in Computer Instrument Systems – Year 2
Bachelor of Science in Applied Physics and Instrumentation – Year 2

**Programme Code:**
**SCIS_8_Y2**
**SPHYS_7_Y2**

**External Examiner(s): Dr. P. Robinson**
**Internal Examiner(s): Dr. M. Brennan**

**Instructions: Anwer Q1 (compulsory) and 2 other questions.**

**Duration:** **2 HOURS**

**Sitting:** Summer 2009

**Requirements for this examination:**

**Note to Candidates:** Please check the Programme Title and the Module Title to ensure that you have received the correct examination paper.
If in doubt please contact an Invigilator.

Q1. (i) Find the probability of obtaining a total of 7 or 11 in a single throw of a pair of dice.

(5 marks)

(ii) A manufacturer has two machines that produce plasma television monitors. Machine I produces 55% of the monitors while machine II produces the remaining 45% of the monitors. Machine I outputs 3% of its run as defective while machine II has 6% of its output defective. Represent this information in a tree diagram. A monitor is found to be defective. Find the probability that this defective monitor was produced by machine II, i.e., determine $P(II|D)$.

(7 marks)

(iii) Use Gaussian elimination to determine $A^{-1}$ where

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ -6 & 2 & 3 \end{bmatrix}$$

(12 marks)

(iv) Determine the mean and the standard deviation using the *assumed mean method* where $x$ represents number of sales and $f$ represents number of salesmen. Hence compute the *coefficient of variation.*

| $x$ | 0-4 | 5-9 | 10-14 | 15-19 | 20-24 | 25-29 |
|---|---|---|---|---|---|---|
| $f$ | 1 | 14 | 23 | 21 | 15 | 6 |

(12 marks)

(v) A quality inspector picks a sample of 10 microchips at random from a very large shipment of microchips known to contain 20% defective microchips. What is the probability that no more than 2 of the microchips picked are defective?

(8 marks)

(vi) An old fashioned string of holiday lights has eight bulbs connected in series. What would have to be the reliability of each bulb if there is to be a 95% chance of the string lighting after a year's storage?

(6 marks)

Q2. (i) Use only the *Gauss Jordan method* to solve the system of linear equations given by,

$$\begin{aligned} x - 2y + 4z &= 1 \\ -x + 3y + z &= -2 \\ 2x - 5y + 2z &= 4. \end{aligned}$$

(10 marks)

(ii) Use *determinants* to decide if the following homogeneous system of linear equations has either the trivial solution or non-trivial solutions.

$$\begin{aligned} 3x - 2y - 2z &= 0 \\ 2x + y - 3z &= 0 \\ x - 6y - z &= 0. \end{aligned}$$

(8 marks)

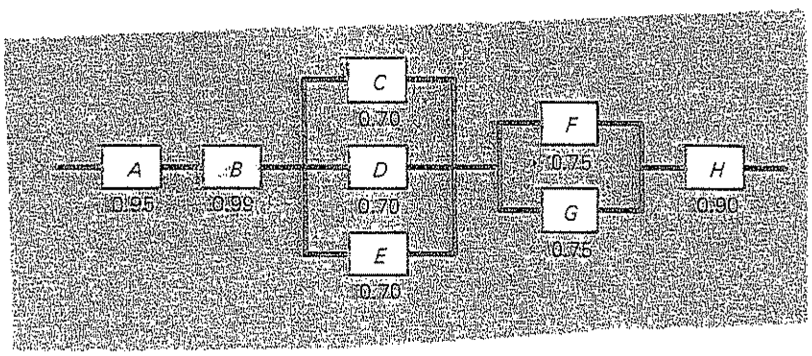
(iii) Given the following row reduced augmented matrix, write down the associated linear system of equations in terms of the variables $x_1, x_2, x_3$ and $x_4$. Identifying the free variable and express the solution set S in terms of the parametric $t$.

$$\left[\begin{array}{cccc|c} 1 & 0 & 6 & 0 & -2 \\ 0 & 1 & -8 & 0 & 5 \\ 0 & 0 & 0 & 1 & 2 \end{array}\right]$$

(7 marks)

Q3. (i) Consider the complex system consisting of eight components arranged in the diagram below. Compute the system reliability showing all steps and intermediate calculations.

(10 marks)



Q3.(ii) A manufacturer estimates that 7% of his output of a small item is defective. Assuming a binomial distribution find the probability that in a sample of 15 items

(a) less than two items are defective,

(b) more than two items will be defective.

(8 marks)

(iii) A component is classified as defective if it has a diameter of less than 70mm. In a batch of 550 components, the mean diameter is 75mm and the standard deviation is 2.8mm. Assuming the diameters are normally distributed, determine how many are likely to be classified as defective.

(7 marks)

Q4. In order to monitor the quality of a production run of steel cylinders, 8 samples, each containing 4 components, are taken at random and their diameter lengths are measured correct to the nearest 0.1mm and tabulated as follows:

| **Sample** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** |
|---|---|---|---|---|---|---|---|---|
| | 102.4 | 102.2 | 99.7 | 99.2 | 101.1 | 101.7 | 100.8 | 103.2 |
| | 99.9 | 100.1 | 100.1 | 99.4 | 101.0 | 99.9 | 100.8 | 100.1 |
| | 101.9 | 101.3 | 102.3 | 100.8 | 102.1 | 99.3 | 102.3 | 97.3 |
| | 100.8 | 101.4 | 100.9 | 99.8 | 101.3 | 100.2 | 101.9 | 99.3 |

(i) Use sample 1 to set up 95% and 99% confidence intervals for the sample mean. Comment briefly on your answers.

(10 Marks)

(ii) Calculate the sample means and ranges. Set up a control chart for the sample means. State, giving reasons, whether or not the process is under control.

(15 Marks)

# Statistical Formulae

Sample mean:

$$\bar{x} = \frac{\Sigma f x}{\Sigma f} = A + c\frac{\Sigma f d}{\Sigma f}, \qquad \text{where } d = \frac{x - A}{c}$$

Population mean:

$$\mu = \frac{\Sigma f x}{\Sigma f} = A + c\frac{\Sigma f d}{\Sigma f}, \qquad \text{where } d = \frac{x - A}{c}$$

Sample standard deviation:

$$s = \sqrt{\frac{\Sigma f (x - \bar{x})^2}{\Sigma f - 1}} = c\sqrt{\frac{\Sigma f d^2}{\Sigma f - 1} - \frac{\Sigma f}{\Sigma f - 1}\left(\frac{\Sigma f d}{\Sigma f}\right)^2}$$

Population standard deviation:

$$\sigma = \sqrt{\frac{\Sigma f (x - \mu)^2}{\Sigma f}} = c\sqrt{\frac{\Sigma f d^2}{\Sigma f} - \left(\frac{\Sigma f d}{\Sigma f}\right)^2}$$

Coefficient of Variation

$$CoV = \frac{s}{\bar{x}} \times 100$$

Binomial distribution:

$$P(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

Poisson distribution:

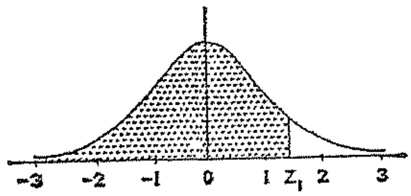
$$P(r) = \frac{\lambda^r e^{-\lambda}}{r!} = e^{-\lambda} \cdot \frac{\lambda^r}{r!}$$

Normal distribution:

$$z = \frac{x - \mu}{\sigma}$$

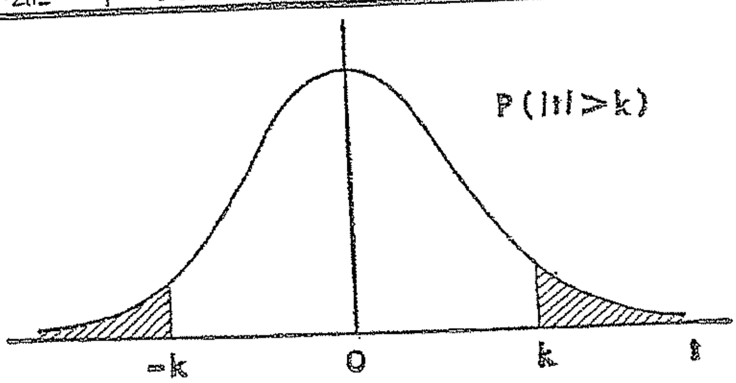
*Achar faoin gCuar Normalach*
*Area under the Normal Curve*

$$P(z \leqslant z_1) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z_1} e^{-\frac{1}{2}z^2}\, dz.$$




| z | 0·00 | 0·01 | 0·02 | 0·03 | 0·04 | 0·05 | 0·06 | 0·07 | 0·08 | 0·09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0·0 | 0·5000 | 5040 | 5080 | 5120 | 5160 | 5199 | 5239 | 5279 | 5319 | 5359 |
| 0·1 | 0·5398 | 5438 | 5478 | 5517 | 5557 | 5596 | 5636 | 5675 | 5714 | 5753 |
| 0·2 | 0·5793 | 5832 | 5871 | 5910 | 5948 | 5987 | 6026 | 6064 | 6103 | 6141 |
| 0·3 | 0·6179 | 6217 | 6255 | 6293 | 6331 | 6368 | 6406 | 6443 | 6480 | 6517 |
| 0·4 | 0·6554 | 6591 | 6628 | 6664 | 6700 | 6736 | 6772 | 6808 | 6844 | 6879 |
| 0·5 | 0·6915 | 6950 | 6985 | 7019 | 7054 | 7088 | 7123 | 7157 | 7190 | 7224 |
| 0·6 | 0·7257 | 7291 | 7324 | 7357 | 7389 | 7422 | 7454 | 7486 | 7517 | 7549 |
| 0·7 | 0·7580 | 7611 | 7642 | 7673 | 7704 | 7734 | 7764 | 7794 | 7823 | 7852 |
| 0·8 | 0·7881 | 7910 | 7939 | 7967 | 7995 | 8023 | 8051 | 8078 | 8106 | 8133 |
| 0·9 | 0·8159 | 8186 | 8212 | 8238 | 8264 | 8289 | 8315 | 8340 | 8365 | 8389 |
| 1·0 | 0·8413 | 8438 | 8461 | 8485 | 8508 | 8531 | 8554 | 8577 | 8599 | 8621 |
| 1·1 | 0·8643 | 8665 | 8686 | 8708 | 8729 | 8749 | 8770 | 8790 | 8810 | 8830 |
| 1·2 | 0·8849 | 8869 | 8888 | 8907 | 8925 | 8944 | 8962 | 8980 | 8997 | 9015 |
| 1·3 | 0·9032 | 9049 | 9066 | 9082 | 9099 | 9115 | 9131 | 9147 | 9162 | 9177 |
| 1·4 | 0·9192 | 9207 | 9222 | 9236 | 9251 | 9265 | 9279 | 9292 | 9306 | 9319 |
| 1·5 | 0·9332 | 9345 | 9357 | 9370 | 9382 | 9394 | 9406 | 9418 | 9429 | 9441 |
| 1·6 | 0·9452 | 9463 | 9474 | 9484 | 9495 | 9505 | 9515 | 9525 | 9535 | 9545 |
| 1·7 | 0·9554 | 9564 | 9573 | 9582 | 9591 | 9599 | 9608 | 9616 | 9625 | 9633 |
| 1·8 | 0·9641 | 9649 | 9656 | 9664 | 9671 | 9678 | 9686 | 9693 | 9699 | 9706 |
| 1·9 | 0·9713 | 9719 | 9726 | 9732 | 9738 | 9744 | 9750 | 9756 | 9761 | 9767 |
| 2·0 | 0·9772 | 9778 | 9783 | 9788 | 9793 | 9798 | 9803 | 9808 | 9812 | 9817 |
| 2·1 | 0·9821 | 9826 | 9830 | 9834 | 9838 | 9842 | 9846 | 9850 | 9854 | 9857 |
| 2·2 | 0·9861 | 9864 | 9868 | 9871 | 9875 | 9878 | 9881 | 9884 | 9887 | 9890 |
| 2·3 | 0·9893 | 9896 | 9898 | 9901 | 9904 | 9906 | 9909 | 9911 | 9913 | 9916 |
| 2·4 | 0·9918 | 9920 | 9922 | 9925 | 9927 | 9929 | 9931 | 9932 | 9934 | 9936 |
| 2·5 | 0·99379 | 99396 | 99413 | 99430 | 99446 | 99461 | 99477 | 99492 | 99506 | 99520 |
| 2·6 | 0·99534 | 99547 | 99560 | 99573 | 99585 | 99598 | 99609 | 99621 | 99632 | 99643 |
| 2·7 | 0·99653 | 99664 | 99674 | 99683 | 99693 | 99702 | 99711 | 99720 | 99728 | 99736 |
| 2·8 | 0·99744 | 99752 | 99760 | 99767 | 99774 | 99781 | 99788 | 99795 | 99801 | 99807 |
| 2·9 | 0·99813 | 99819 | 99825 | 99831 | 99836 | 99841 | 99846 | 99851 | 99856 | 99861 |
| 3·0 | 0·99865 | 99869 | 99874 | 99878 | 99882 | 99886 | 99889 | 99893 | 99897 | 99900 |
| 3·1 | 0·99903 | 99906 | 99910 | 99913 | 99916 | 99918 | 99921 | 99924 | 99926 | 99929 |
| 3·2 | 0·99931 | 99934 | 99936 | 99938 | 99940 | 99942 | 99944 | 99946 | 99948 | 99950 |
| 3·3 | 0·99952 | 99953 | 99955 | 99957 | 99958 | 99960 | 99961 | 99962 | 99964 | 99965 |
| 3·4 | 0·99966 | 99968 | 99969 | 99970 | 99971 | 99972 | 99973 | 99974 | 99975 | 99976 |
| 3·5 | 0·99977 | 99978 | 99978 | 99979 | 99980 | 99981 | 99981 | 99982 | 99983 | 99983 |
| 3·6 | 0·99984 | 99985 | 99985 | 99986 | 99986 | 99987 | 99987 | 99988 | 99988 | 99989 |
| 3·7 | 0·99989 | 99990 | 99990 | 99990 | 99991 | 99991 | 99992 | 99992 | 99992 | 99992 |
| 3·8 | 0·99993 | 99993 | 99993 | 99994 | 99994 | 99994 | 99994 | 99995 | 99995 | 99995 |
| 3·9 | 0·99995 | 99995 | 99996 | 99996 | 99996 | 99996 | 99996 | 99996 | 99997 | 99997 |

| *t*-DÁILEADH | | | | | | *t*-DISTRIBUTION |
|---|---|---|---|---|---|---|
| n − 1 | **20** | **10** | **5** | **2** | **1** | **0·2** |
| 1 | 3·078 | 6·314 | 12·706 | 31·821 | 63·657 | 318·310 |
| 2 | 1·886 | 2·920 | 4·303 | 6·965 | 9·925 | 22·327 |
| 3 | 1·638 | 2·353 | 3·182 | 4·541 | 5·841 | 10·215 |
| 4 | 1·533 | 2·132 | 2·776 | 3·747 | 4·604 | 7·173 |
| 5 | 1·476 | 2·015 | 2·571 | 3·365 | 4·032 | 5·893 |
| 6 | 1·440 | 1·943 | 2·447 | 3·143 | 3·707 | 5·208 |
| 7 | 1·415 | 1·895 | 2·365 | 2·998 | 3·499 | 4·785 |
| 8 | 1·397 | 1·860 | 2·306 | 2·896 | 3·355 | 4·501 |
| 9 | 1·383 | 1·833 | 2·262 | 2·821 | 3·250 | 4·297 |
| **10** | 1·372 | 1·812 | 2·228 | 2·764 | 3·169 | 4·144 |
| 11 | 1·363 | 1·796 | 2·201 | 2·718 | 3·106 | 4·025 |
| 12 | 1·356 | 1·782 | 2·179 | 2·681 | 3·055 | 3·930 |
| 13 | 1·350 | 1·771 | 2·160 | 2·650 | 3·012 | 3·852 |
| 14 | 1·345 | 1·761 | 2·145 | 2·624 | 2·977 | 3·787 |
| 15 | 1·341 | 1·753 | 2·131 | 2·602 | 2·947 | 3·733 |
| 16 | 1·337 | 1·746 | 2·120 | 2·583 | 2·921 | 3·686 |
| 17 | 1·333 | 1·740 | 2·110 | 2·567 | 2·898 | 3·646 |
| 18 | 1·330 | 1·734 | 2·101 | 2·552 | 2·878 | 3·610 |
| 19 | 1·328 | 1·729 | 2·093 | 2·539 | 2·861 | 3·579 |
| **20** | 1·325 | 1·725 | 2·086 | 2·528 | 2·845 | 3·552 |
| 21 | 1·323 | 1·721 | 2·080 | 2·518 | 2·831 | 3·527 |
| 22 | 1·321 | 1·717 | 2·074 | 2·508 | 2·819 | 3·505 |
| 23 | 1·319 | 1·714 | 2·069 | 2·500 | 2·807 | 3·485 |
| 24 | 1·318 | 1·711 | 2·064 | 2·492 | 2·797 | 3·467 |
| 25 | 1·316 | 1·708 | 2·060 | 2·485 | 2·787 | 3·450 |
| 26 | 1·315 | 1·706 | 2·056 | 2·479 | 2·779 | 3·435 |
| 27 | 1·314 | 1·703 | 2·052 | 2·473 | 2·771 | 3·421 |
| 28 | 1·313 | 1·701 | 2·048 | 2·467 | 2·763 | 3·408 |
| 29 | 1·311 | 1·699 | 2·045 | 2·462 | 2·756 | 3·396 |
| **30** | 1·310 | 1·697 | 2·042 | 2·457 | 2·750 | 3·385 |
| **40** | 1·303 | 1·684 | 2·021 | 2·423 | 2·704 | 3·307 |
| **60** | 1·296 | 1·671 | 2·000 | 2·390 | 2·660 | 3·232 |
| **120** | 1·289 | 1·658 | 1·980 | 2·358 | 2·617 | 3·160 |
| ∞ | 1·282 | 1·645 | 1·960 | 2·326 | 2·576 | 3·090 |



# Control Chart Coefficients

Table 1

| $n$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|
| $a_n$ | 0.8862 | 0.5908 | 0.4857 | 0.4299 | 0.3946 | 0.3698 | 0.3512 | 0.3367 |

Table 2

| Sample Size $n$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $A_{0.025}$ | 1.229 | 0.608 | 0.476 | 0.377 | 0.316 | 0.274 | 0.244 | 0.202 | 0.220 | 0.186 | 0.174 |
| $A_{0.001}$ | 1.937 | 1.054 | 0.750 | 0.594 | 0.498 | 0.432 | 0.384 | 0.347 | 0.317 | 0.294 | 0.274 |

Table 3

| | *For use with* $\sigma$ | | | | *For use with* $\overline{w}$ | | | |
|---|---|---|---|---|---|---|---|---|
| $n$ | $D_{0.001}$ | $F_{0.025}$ | $F_{0.975}$ | $D_{0.999}$ | $D'_{0.001}$ | $F'_{0.025}$ | $F'_{0.975}$ | $D'_{0.999}$ |
| 2 | 0.00 | 0.04 | 3.17 | 4.65 | 0.00 | 0.04 | 2.81 | 4.12 |
| 3 | 0.06 | 0.30 | 3.68 | 5.06 | 0.04 | 0.18 | 2.17 | 2.99 |
| 4 | 0.20 | 0.30 | 3.98 | 5.31 | 0.10 | 0.29 | 1.93 | 2.58 |
| 5 | 0.37 | 0.85 | 4.20 | 5.48 | 0.16 | 0.37 | 1.81 | 2.36 |
| 6 | 0.54 | 1.06 | 4.36 | 5.62 | 0.20 | 0.42 | 1.72 | 2.22 |
| 7 | 0.69 | 1.25 | 4.49 | 5.73 | 0.26 | 0.46 | 1.66 | 2.12 |
| 8 | 0.83 | 1.41 | 4.61 | 5.82 | 0.29 | 0.50 | 1.62 | 2.04 |
| 9 | 0.96 | 1.55 | 4.70 | 5.90 | 0.32 | 0.52 | 1.58 | 1.99 |
| 10 | 1.08 | 1.67 | 4.79 | 5.97 | 0.35 | 0.54 | 1.56 | 1.94 |
| 11 | 1.20 | 1.78 | 4.86 | 6.04 | 0.38 | 0.56 | 1.53 | 1.90 |
| 12 | 1.30 | 1.88 | 4.92 | 6.09 | 0.40 | 0.58 | 1.51 | 1.87 |