CORK INSTITUTE OF TECHNOLOGY
INSTITIÚID TEICNEOLAÍOCHTA CHORCAÍ

Autumn Examinations 2011

Module Title: Mathematics for Science 2.2 with Maple

Module Code: MATH 6038

School: Science & Informatics

Programme Title: Bachelor of Science in Applied Physics & Instrumentation – Year 2
Higher Certificate in Industrial Measurement & Control – Year 2

Programme Code: SPHYS_7_Y2
SIMCT_6_Y2

External Examiner(s): Dr. P. Robinson

Internal Examiner(s): Dr. M. Brennan

Instructions: Answer Q1 (COMPUSORY) and any other 2 questions.
Q1 carries 50 marks. All other questions carry 25 marks.

Duration: 2 Hours

Sitting: Autumn 2011

Requirements for this examination: Mathematical Tables

Note to Candidates: Please check the Programme Title and the Module Title to ensure that you have received the correct examination.
If in doubt please contact an Invigilator

Q1. (a) Determine A^{-1} where $A = \begin{bmatrix} 1 & 1 & -1 \\ -3 & 2 & -1 \\ 3 & -3 & 2 \end{bmatrix}$

(10 marks)

(b) Wires manufactured for use in a certain electronic device are specificed to have resistances between $0.16\,\Omega$ and $0.18\,\Omega$. The actual measured resistances of the wires have a *normal* distribution with a mean of $0.17\,\Omega$ and a standard deviation of $0.005\,\Omega$. What is the probability that a randomly selected wire will meet the specifications?

(8 marks)

(c) Mobile Phones from the CIT shop come in three varieties, *Samsung*, *Nokia* or *iPhone*. Of all such mobiles, 25% are *Samsung*, 35% *Nokia*, and 40% *iPhone*. Further it is known that 3% of all *Samsung* mobiles are defective, 1% of *Nokia* are defective, and 2% of *iPhone* are defective.

(i) Represent this information in a tree diagram.

(ii) Find the probability that a mobile chosen at random is a *Nokia* or an *iPhone*.

(iii) Find the probability that a mobile chosen at random is defective.

(iv) A certain mobile is found to be defective in the CIT shop.
Find the probability that this defective calculator is a *Nokia*.

(12 marks)

(d) The number of vehicle failures satisfies a *Poisson* distribution. Over the year the number of failures of a fleet of vehicles for each month is given by the following table.

J	F	M	A	M	J	J	A	S	O	N	D
3	1	0	1	0	1	2	3	1	0	1	2

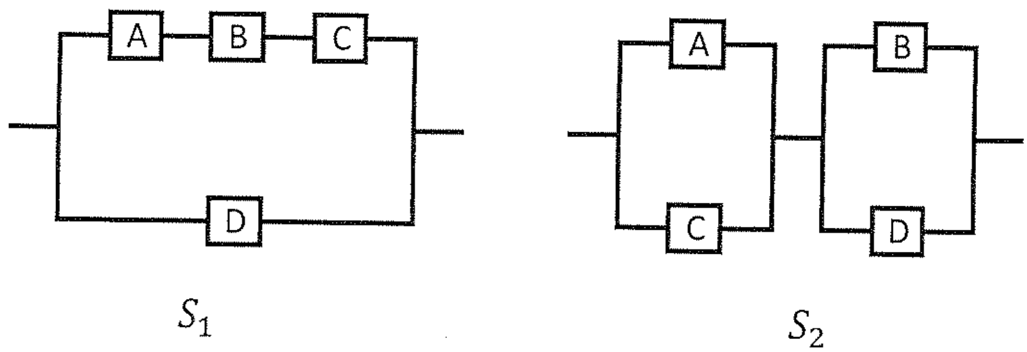
(i) What is the probability of 2 failures in a given month.

(ii) What is the probability of at most 2 failures in a given month.

(8 marks)

Q1. (e) A *Reliability Block Diagram* is given for two systems S_1 and S_2. Determine $P(S_1)$ and $P(S_2)$ and hence identify the most reliable system where the reliabilities P(A)=0.9, P(B)=0.7 P(C)=0.8 and P(D)=0.9. Carefully show all steps and intermediate calculations.

(12 marks)



Q2. (a) Apply **only** the *Gauss-Jordan Method* to solve the system of linear equations

$$\begin{aligned} -x + y + z &= -3 \\ -2x + 4y + z &= 2 \\ 2x - 3y - z &= 1. \end{aligned}$$

(ii) Verify y using *Cramer's Rule.*

(13 marks)

(b) It is assumed that the weight of goods packed by a certain machine are *normally* distributed with a mean weight of $8kg$ and a standard deviation of $0.03kg$. Calculate the probability that a package taken at random will weigh

(i) less than $8.07kg$

(ii) greater than $8.08kg$

(iii) between than $7.98kg$ and $8.05kg$?

If 99.8% of readings are less than some critical weight W, find the value of W.

(12 marks)

Q3.(a) A sample of 80 ball bearings was taken from the production of machine A and their diameters (in cm) where measured to give the following distribution.

x	0.80-0.82	0.83-0.85	0.86-0.88	0.89-0.91	0.92-0.94	0.95-0.97	0.98-1.00
f	6	8	15	23	18	6	4

(i) Find the *mode* for the above distribution.

(ii) Use the *assumed mean method* to determine the mean and standard deviation.

(iii) A second sample of 100 ball bearings was taken from machine B, giving a mean diameter of $0.73cm$ with standard deviation of $0.04cm$. Compute the *coefficient of variation* for each machine. Which machine has the greater variation?

(18 marks)

(b) Suppose a large lot contains 8% defective fuses. Assuming a *binomial* distribution find the probability that in a sample of 12 fuses, either five or six fuses are defective.

(7 marks)

Q4. In order to monitor the quality of a production run of aluminium bolts, 8 samples, each containing 4 components, are taken at random and their diameter lengths are measured correct to the nearest 0.1mm and tabulated as follows:

Sample	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
	88.3	91.1	87.6	89.2	81.2	91.7	81.8	90.2
	91.0	91.2	85.2	89.4	82.0	89.9	81.8	90.2
	91.9	89.3	82.3	90.8	82.1	88.2	90.3	87.3
	90.8	93.4	92.9	86.8	81.3	80.2	81.9	89.3
Means $\overline{x}_i$	$\overline{x}_1$	$\overline{x}_2$	$\overline{x}_3$	$\overline{x}_4$	81.65	87.5	83.95	89.25
Ranges w_i	w_1	w_2	w_3	w_4	0.9	11.5	8.5	2.9

(a) Use sample 3 to set up 95% and 99% confidence intervals for the population mean. Comment briefly on your answers.

(10 Marks)

(b) Calculate the remaining sample means $\overline{x}_i$ and ranges w_i. Find the grand mean $\overline{\overline{x}}$ and the associated *outer* and *inner control limits*. Hence set up a control chart for the sample means. State, giving reasons, whether or not the process is under control.

(15 Marks)

Statistical Formulae

Sample mean:

$$\bar{x} = \frac{\Sigma f x}{\Sigma f} = A + c\frac{\Sigma f d}{\Sigma f}, \qquad \text{where } d = \frac{x - A}{c}$$

Population mean:

$$\mu = \frac{\Sigma f x}{\Sigma f} = A + c\frac{\Sigma f d}{\Sigma f}, \qquad \text{where } d = \frac{x - A}{c}$$

Sample standard deviation:

$$s = \sqrt{\frac{\Sigma f(x - \bar{x})^2}{\Sigma f - 1}} = c\sqrt{\frac{\Sigma f d^2}{\Sigma f - 1} - \frac{\Sigma f}{\Sigma f - 1}\left(\frac{\Sigma f d}{\Sigma f}\right)^2}$$

Population standard deviation:

$$\sigma = \sqrt{\frac{\Sigma f(x - \mu)^2}{\Sigma f}} = c\sqrt{\frac{\Sigma f d^2}{\Sigma f} - \left(\frac{\Sigma f d}{\Sigma f}\right)^2}$$

Coefficient of Variation

$$\text{CoV} = \frac{s}{\bar{x}} \times 100 \qquad \Big| \qquad \text{Mode} = L + \left(\frac{l}{l+u}\right)c$$

Binomial distribution:

$$P(k) = \binom{n}{k} p^k (1 - p)^{n-k}$$

Poisson distribution:

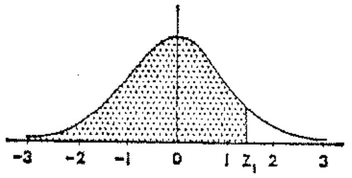
$$P(r) = \frac{\lambda^r e^{-\lambda}}{r!} = e^{-\lambda} \cdot \frac{\lambda^r}{r!}$$

Normal distribution:

$$z = \frac{x - \mu}{\sigma}$$

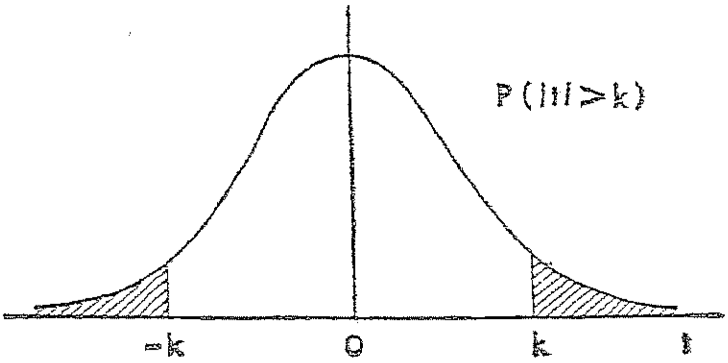
Normal Distribution Tables

Area under the Normal Curve $P(z \le z_1) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z_1} e^{-\frac{z^2}{2}} dz$



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7703	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.99977	0.99978	0.99978	0.99979	0.99980	0.99981	0.99981	0.99982	0.99983	0.99983
3.6	0.99984	0.99985	0.99985	0.99986	0.99986	0.99987	0.99987	0.99988	0.99988	0.99989
3.7	0.99989	0.99990	0.99990	0.99990	0.99991	0.99991	0.99992	0.99992	0.99992	0.99992
3.8	0.99993	0.99993	0.99993	0.99994	0.99994	0.99994	0.99994	0.99995	0.99995	0.99995
3.9	0.99995	0.99995	0.99996	0.99996	0.99996	0.99996	0.99996	0.99996	0.99997	0.99997

t-DÁILEADH						t-DISTRIBUTION
n − 1	20	10	5	2	1	0·2
1	3·078	6·314	12·706	31·821	63·657	318·310
2	1·886	2·920	4·303	6·965	9·925	22·327
3	1·638	2·353	3·182	4·541	5·841	10·215
4	1·533	2·132	2·776	3·747	4·604	7·173
5	1·476	2·015	2·571	3·365	4·032	5·893
6	1·440	1·943	2·447	3·143	3·707	5·208
7	1·415	1·895	2·365	2·998	3·499	4·785
8	1·397	1·860	2·306	2·896	3·355	4·501
9	1·383	1·833	2·262	2·821	3·250	4·297
10	1·372	1·812	2·228	2·764	3·169	4·144
11	1·363	1·796	2·201	2·718	3·106	4·025
12	1·356	1·782	2·179	2·681	3·055	3·930
13	1·350	1·771	2·160	2·650	3·012	3·852
14	1·345	1·761	2·145	2·624	2·977	3·787
15	1·341	1·753	2·131	2·602	2·947	3·733
16	1·337	1·746	2·120	2·583	2·921	3·686
17	1·333	1·740	2·110	2·567	2·898	3·646
18	1·330	1·734	2·101	2·552	2·878	3·610
19	1·328	1·729	2·093	2·539	2·861	3·579
20	1·325	1·725	2·086	2·528	2·845	3·552
21	1·323	1·721	2·080	2·518	2·831	3·527
22	1·321	1·717	2·074	2·508	2·819	3·505
23	1·319	1·714	2·069	2·500	2·807	3·485
24	1·318	1·711	2·064	2·492	2·797	3·467
25	1·316	1·708	2·060	2·485	2·787	3·450
26	1·315	1·706	2·056	2·479	2·779	3·435
27	1·314	1·703	2·052	2·473	2·771	3·421
28	1·313	1·701	2·048	2·467	2·763	3·408
29	1·311	1·699	2·045	2·462	2·756	3·396
30	1·310	1·697	2·042	2·457	2·750	3·385
40	1·303	1·684	2·021	2·423	2·704	3·307
60	1·296	1·671	2·000	2·390	2·660	3·232
120	1·289	1·658	1·980	2·358	2·617	3·160
∞	1·282	1·645	1·960	2·326	2·576	3·090



Control Chart Coefficients

Table 1

n	2	3	4	5	6	7	8	9
a_n	0.8862	0.5908	0.4857	0.4299	0.3946	0.3698	0.3512	0.3367

Table 2

Sample Size n	2	3	4	5	6	7	8	9	10	11	12
$A_{0.025}$	1.229	0.608	0.476	0.377	0.316	0.274	0.244	0.202	0.220	0.186	0.174
$A_{0.001}$	1.937	1.054	0.750	0.594	0.498	0.432	0.384	0.347	0.317	0.294	0.274

Table 3

	For use with σ				For use with $\overline{w}$			
n	$D_{0.001}$	$F_{0.025}$	$F_{0.975}$	$D_{0.999}$	$D'_{0.001}$	$F'_{0.025}$	$F'_{0.975}$	$D'_{0.999}$
2	0.00	0.04	3.17	4.65	0.00	0.04	2.81	4.12
3	0.06	0.30	3.68	5.06	0.04	0.18	2.17	2.99
4	0.20	0.30	3.98	5.31	0.10	0.29	1.93	2.58
5	0.37	0.85	4.20	5.48	0.16	0.37	1.81	2.36
6	0.54	1.06	4.36	5.62	0.20	0.42	1.72	2.22
7	0.69	1.25	4.49	5.73	0.26	0.46	1.66	2.12
8	0.83	1.41	4.61	5.82	0.29	0.50	1.62	2.04
9	0.96	1.55	4.70	5.90	0.32	0.52	1.58	1.99
10	1.08	1.67	4.79	5.97	0.35	0.54	1.56	1.94
11	1.20	1.78	4.86	6.04	0.38	0.56	1.53	1.90
12	1.30	1.88	4.92	6.09	0.40	0.58	1.51	1.87