CORK INSTITUTE OF TECHNOLOGY
INSTITIÚID TEICNEOLAÍOCHTA CHORCAÍ

Autumn Examinations 2009

Module Title: Mathematics for Science 2.2 with Maple

Module Code: **MATH 6038**

School: School of Science

Programme Title:
Bachelor of Science (Honours) in Computer Instrument Systems – Year 2
Bachelor of Science in Applied Physics and Instrumentation – Year 2

Programme Code:
SCIS_8_Y2
SPHYS_7_Y2

External Examiner(s): Dr. P. Robinson
Internal Examiner(s): Dr. M. Brennan

Instructions: Anwer Q1 (compulsory) and 2 other questions.

Duration: **2 HOURS**

Sitting: Autumn 2009

Requirements for this examination:

Note to Candidates: Please check the Programme Title and the Module Title to ensure that you have received the correct examination paper.
If in doubt please contact an Invigilator.

Q1. (i) Find the probability of obtaining a total of 4 or 12 in a single throw of a pair of dice.

(5 marks)

(ii) A new diagnostic test gives a positive indication 95% of the time when an individual has the disease X, and a negative response 90% of the time when an individual does not have the disease X. It is estimated that 10% of the population suffer from disease X.

(a) Draw a tree diagram to represent the above information.

(b) A person selected at random reacts positively to the test. What is the probability that this person actually has disease X?

(c) Calculate the probability that the test yields the correct diagnosis for an individual chosen at random.

(11 marks)

(iii) Use only the *Gauss Jordan method* to solve the system of linear equations given by,

$$\begin{array}{rcrcrcr} x & - & 2y & + & 4z & = & 4 \\ -3x & + & 9y & + & 3z & = & -12 \\ 2x & - & 5y & + & 2z & = & 4. \end{array}$$

Verify the solution.

(12 marks)

(iv) Construct a histogram using upper and lower class boundaries. Determine the mean and mode for the following frequency distribution where x represents the gain and f represents number of transistors.

x	83.5–85.5	86.5–88.5	89.5–91.5	92.5–94.5	95.5–97.5	98.5–100.5
f	6	39	26	15	3	1

(10 marks)

Q1.ctd (v) The probability that a car will not develop a major fault within the first 3 years of its life is 0.997. Assuming a binomial distribution, find the probability that out of 20 cars selected at random,

(a) 19 will not develop any major faults in the first 3 years,

(b) 19 or more will not develop any major faults in the first 3 years.

(8 marks)

(vi) A torch consists of a battery, a switch and a bulb, assembled in series. The reliabilities of the battery, switch and bulb are 0.9, 0.8 and 0.7 respectively. Calculate the system reliability

(4 marks)

Q2. (i) Use Gaussian elimination to determine A^{-1} where

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & -1 & 4 \\ -3 & -1 & 2 \end{bmatrix}$$

(10 marks)

(ii) Use *determinants* to decide if the following homogeneous system of linear equations has either the trivial solution or non-trivial solutions.

$$\begin{aligned} 3x - 4y - 2z &= 0 \\ 2x + y - 3z &= 0 \\ 2x - 5y - z &= 0. \end{aligned}$$

(8 marks)

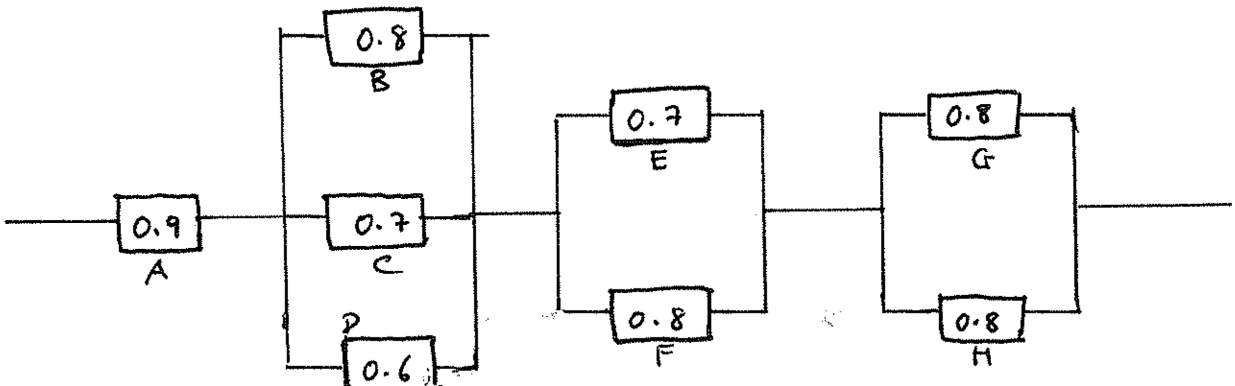
(iii) Given the following row reduced augmented matrix, write down the associated linear system of equations in terms of the variables x_1, x_2, x_3 and x_4. Identifying the free variable and express the solution set S in terms of the parametric t.

$$\left[\begin{array}{cccc|c} 1 & 0 & 8 & 0 & -4 \\ 0 & 1 & -2 & 0 & 9 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right]$$

(7 marks)

Q3. (i) Consider the complex system consisting of eight components arranged in the diagram below. Compute the system reliability showing *all steps and intermediate calculations.*

(9 marks)



(ii) The probability that a hard drive fails in any month is 0.004. An engineer is responsible for 500 drives. Assuming a poisson distribution, calculate the probability that in a months, the number of drives failing is

(a) none (b) one, (c) more than two.

(8 marks)

(iii) The mean diameter of holes produced by a drilling machine bit is 4.05mm and the standard deviation of the diameters is 0.0038mm. For twenty holes drilled using this machine, determine, correct to nearest whole number, how many are likely to have diameters of between,

(a) 4.048 and 4.0553mm (b) 4.052 and 4.056mm

assuming the diameters are normally distributed.

(8 marks)

Q4. In order to monitor the quality of a production run of steel cylinders, 8 samples, each containing 4 components, are taken at random and their diameter lengths are measured correct to the nearest 0.1mm and tabulated as follows:

Sample	1	2	3	4	5	6	7	8
	92.4	92.2	89.7	89.2	91.1	91.7	90.8	93.2
	89.9	90.1	90.1	89.4	91.0	89.9	90.8	90.1
	91.9	91.3	92.3	90.8	92.1	89.3	92.3	87.3
	90.8	91.4	90.9	89.8	91.3	80.2	91.9	89.3

(i) Use sample 5 to set up 95% and 99% confidence intervals for the sample mean. Comment briefly on your answers.

(10 Marks)

(ii) Calculate the sample means and ranges. Set up a control chart for the sample means. State, giving reasons, whether or not the process is under control.

(15 Marks)

Statistical Formulae

Sample mean:

$$\bar{x} = \frac{\Sigma f x}{\Sigma f} = A + c\frac{\Sigma f d}{\Sigma f}, \qquad \text{where } d = \frac{x-A}{c}$$

Population mean:

$$\mu = \frac{\Sigma f x}{\Sigma f} = A + c\frac{\Sigma f d}{\Sigma f}, \qquad \text{where } d = \frac{x-A}{c}$$

Sample standard deviation:

$$s = \sqrt{\frac{\Sigma f(x-\bar{x})^2}{\Sigma f - 1}} = c\sqrt{\frac{\Sigma f d^2}{\Sigma f - 1} - \frac{\Sigma f}{\Sigma f - 1}\left(\frac{\Sigma f d}{\Sigma f}\right)^2}$$

Population standard deviation:

$$\sigma = \sqrt{\frac{\Sigma f(x-\mu)^2}{\Sigma f}} = c\sqrt{\frac{\Sigma f d^2}{\Sigma f} - \left(\frac{\Sigma f d}{\Sigma f}\right)^2}$$

Coefficient of Variation

$$CoV = \frac{s}{\bar{x}} \times 100$$

Binomial distribution:

$$P(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

Poisson distribution:

$$P(r) = \frac{\lambda^r e^{-\lambda}}{r!} = e^{-\lambda} \cdot \frac{\lambda^r}{r!}$$

Normal distribution:

$$z = \frac{x-\mu}{\sigma}$$

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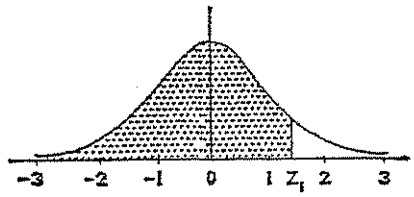
Area under the Normal Curve

$$P(z \leqslant z_1) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z_1} e^{-\frac{1}{2}z^2}\, dz .$$



z	0·00	0·01	0·02	0·03	0·04	0·05	0·06	0·07	0·08	0·09
0·0	0·5000	5040	5080	5120	5160	5199	5239	5279	5319	5359
0·1	0·5398	5438	5478	5517	5557	5596	5636	5675	5714	5753
0·2	0·5793	5832	5871	5910	5948	5987	6026	6064	6103	6141
0·3	0·6179	6217	6255	6293	6331	6368	6406	6443	6480	6517
0·4	0·6554	6591	6628	6664	6700	6736	6772	6808	6844	6879
0·5	0·6915	6950	6985	7019	7054	7088	7123	7157	7190	7224
0·6	0·7257	7291	7324	7357	7389	7422	7454	7486	7517	7549
0·7	0·7580	7611	7642	7673	7704	7734	7764	7794	7823	7852
0·8	0·7881	7910	7939	7967	7995	8023	8051	8078	8106	8133
0·9	0·8159	8186	8212	8238	8264	8289	8315	8340	8365	8389
1·0	0·8413	8438	8461	8485	8508	8531	8554	8577	8599	8621
1·1	0·8643	8665	8686	8708	8729	8749	8770	8790	8810	8830
1·2	0·8849	8869	8888	8907	8925	8944	8962	8980	8997	9015
1·3	0·9032	9049	9066	9082	9099	9115	9131	9147	9162	9177
1·4	0·9192	9207	9222	9236	9251	9265	9279	9292	9306	9319
1·5	0·9332	9345	9357	9370	9382	9394	9406	9418	9429	9441
1·6	0·9452	9463	9474	9484	9495	9505	9515	9525	9535	9545
1·7	0·9554	9564	9573	9582	9591	9599	9608	9616	9625	9633
1·8	0·9641	9649	9656	9664	9671	9678	9686	9693	9699	9706
1·9	0·9713	9719	9726	9732	9738	9744	9750	9756	9761	9767
2·0	0·9772	9778	9783	9788	9793	9798	9803	9808	9812	9817
2·1	0·9821	9826	9830	9834	9838	9842	9846	9850	9854	9857
2·2	0·9861	9864	9868	9871	9875	9878	9881	9884	9887	9890
2·3	0·9893	9896	9898	9901	9904	9906	9909	9911	9913	9916
2·4	0·9918	9920	9922	9925	9927	9929	9931	9932	9934	9936
2·5	0·99379	99396	99413	99430	99446	99461	99477	99492	99506	99520
2·6	0·99534	99547	99560	99573	99585	99598	99609	99621	99632	99643
2·7	0·99653	99664	99674	99683	99693	99702	99711	99720	99728	99736
2·8	0·99744	99752	99760	99767	99774	99781	99788	99795	99801	99807
2·9	0·99813	99819	99825	99831	99836	99841	99846	99851	99856	99861
3·0	0·99865	99869	99874	99878	99882	99886	99889	99893	99897	99900
3·1	0·99903	99906	99910	99913	99916	99918	99921	99924	99926	99929
3·2	0·99931	99934	99936	99938	99940	99942	99944	99946	99948	99950
3·3	0·99952	99953	99955	99957	99958	99960	99961	99962	99964	99965
3·4	0·99966	99968	99969	99970	99971	99972	99973	99974	99975	99976
3·5	0·99977	99978	99978	99979	99980	99981	99981	99982	99983	99983
3·6	0·99984	99985	99985	99986	99986	99987	99987	99988	99988	99989
3·7	0·99989	99990	99990	99990	99991	99991	99992	99992	99992	99992
3·8	0·99993	99993	99993	99994	99994	99994	99994	99995	99995	99995
3·9	0·99995	99995	99996	99996	99996	99996	99996	99996	99997	99997

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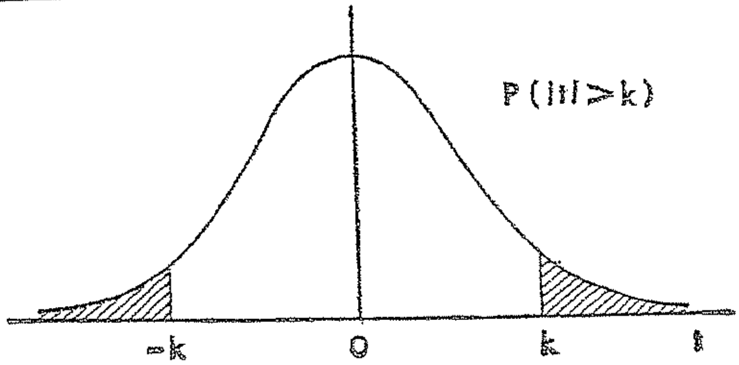
n − 1	20	10	5	2	1	0·2
1	3·078	6·314	12·706	31·821	63·657	318·310
2	1·886	2·920	4·303	6·965	9·925	22·327
3	1·638	2·353	3·182	4·541	5·841	10·215
4	1·533	2·132	2·776	3·747	4·604	7·173
5	1·476	2·015	2·571	3·365	4·032	5·893
6	1·440	1·943	2·447	3·143	3·707	5·208
7	1·415	1·895	2·365	2·998	3·499	4·785
8	1·397	1·860	2·306	2·896	3·355	4·501
9	1·383	1·833	2·262	2·821	3·250	4·297
10	1·372	1·812	2·228	2·764	3·169	4·144
11	1·363	1·796	2·201	2·718	3·106	4·025
12	1·356	1·782	2·179	2·681	3·055	3·930
13	1·350	1·771	2·160	2·650	3·012	3·852
14	1·345	1·761	2·145	2·624	2·977	3·787
15	1·341	1·753	2·131	2·602	2·947	3·733
16	1·337	1·746	2·120	2·583	2·921	3·686
17	1·333	1·740	2·110	2·567	2·898	3·646
18	1·330	1·734	2·101	2·552	2·878	3·610
19	1·328	1·729	2·093	2·539	2·861	3·579
20	1·325	1·725	2·086	2·528	2·845	3·552
21	1·323	1·721	2·080	2·518	2·831	3·527
22	1·321	1·717	2·074	2·508	2·819	3·505
23	1·319	1·714	2·069	2·500	2·807	3·485
24	1·318	1·711	2·064	2·492	2·797	3·467
25	1·316	1·708	2·060	2·485	2·787	3·450
26	1·315	1·706	2·056	2·479	2·779	3·435
27	1·314	1·703	2·052	2·473	2·771	3·421
28	1·313	1·701	2·048	2·467	2·763	3·408
29	1·311	1·699	2·045	2·462	2·756	3·396
30	1·310	1·697	2·042	2·457	2·750	3·385
40	1·303	1·684	2·021	2·423	2·704	3·307
60	1·296	1·671	2·000	2·390	2·660	3·232
120	1·289	1·658	1·980	2·358	2·617	3·160
∞	1·282	1·645	1·960	2·326	2·576	3·090



Control Chart Coefficients

Table 1

n	2	3	4	5	6	7	8	9
a_n	0.8862	0.5908	0.4857	0.4299	0.3946	0.3698	0.3512	0.3367

Table 2

Sample Size n	2	3	4	5	6	7	8	9	10	11	12
$A_{0.025}$	1.229	0.608	0.476	0.377	0.316	0.274	0.244	0.202	0.220	0.186	0.174
$A_{0.001}$	1.937	1.054	0.750	0.594	0.498	0.432	0.384	0.347	0.317	0.294	0.274

Table 3

	For use with σ				*For use with* $\overline{w}$			
n	$D_{0.001}$	$F_{0.025}$	$F_{0.975}$	$D_{0.999}$	$D'_{0.001}$	$F'_{0.025}$	$F'_{0.975}$	$D'_{0.999}$
2	0.00	0.04	3.17	4.65	0.00	0.04	2.81	4.12
3	0.06	0.30	3.68	5.06	0.04	0.18	2.17	2.99
4	0.20	0.30	3.98	5.31	0.10	0.29	1.93	2.58
5	0.37	0.85	4.20	5.48	0.16	0.37	1.81	2.36
6	0.54	1.06	4.36	5.62	0.20	0.42	1.72	2.22
7	0.69	1.25	4.49	5.73	0.26	0.46	1.66	2.12
8	0.83	1.41	4.61	5.82	0.29	0.50	1.62	2.04
9	0.96	1.55	4.70	5.90	0.32	0.52	1.58	1.99
10	1.08	1.67	4.79	5.97	0.35	0.54	1.56	1.94
11	1.20	1.78	4.86	6.04	0.38	0.56	1.53	1.90
12	1.30	1.88	4.92	6.09	0.40	0.58	1.51	1.87