MATH6014: Test 1 B (08/03/11)

Name:

Answer all questions. Marks may be lost if necessary work is not clearly shown.

Question 1

(a) Solve for x:

$$\frac{8^{x+2}2^{-x}}{(\sqrt{2})^4} = \frac{1}{8}.$$

(b) Let

$$f(x) = \log_e e^{101} + 4\log_e 2x + \log_e \sqrt{x} - 2\log_e 4 - \frac{9}{2}\log_e x.$$

This function is defined for all $x > 0$. By invoking the log rules and simplifying, show that $f(x)$ is an integer for all $x > 0$ (i.e. $f(x) \in \mathbb{Z}$.).

Solution

(a)

(b)

Question 2

(a) Suppose that x and y are real numbers such that $x+y=54$ and $x \neq y$. Simplify

$$\frac{x^2-y^2}{5(x-y)+6(x-y)+7(x-y)}.$$

Now, in addition to $x+y=54$ and $x \neq y$, let $z, w \in \mathbb{R}$ such that $xw-yz \neq 0$. By using the distributive law twice, factorise.

$$x^2w-xyz+xyw-y^2z$$

Hence, use the no zero divisors theorem to show that $x^2w-xyz+xyw-y^2z$ is non-zero.

(b) Solve for a:

$$b^3=\sqrt{\frac{1+a}{1-a}}.$$

Solution

(a)

(b)

Question 3

(a) Consider the following lines defined on the plane:

A. $y = x + 1$.

B. $x = 2$.

C. $y = 3$.

D. $y = -3x - 1$.

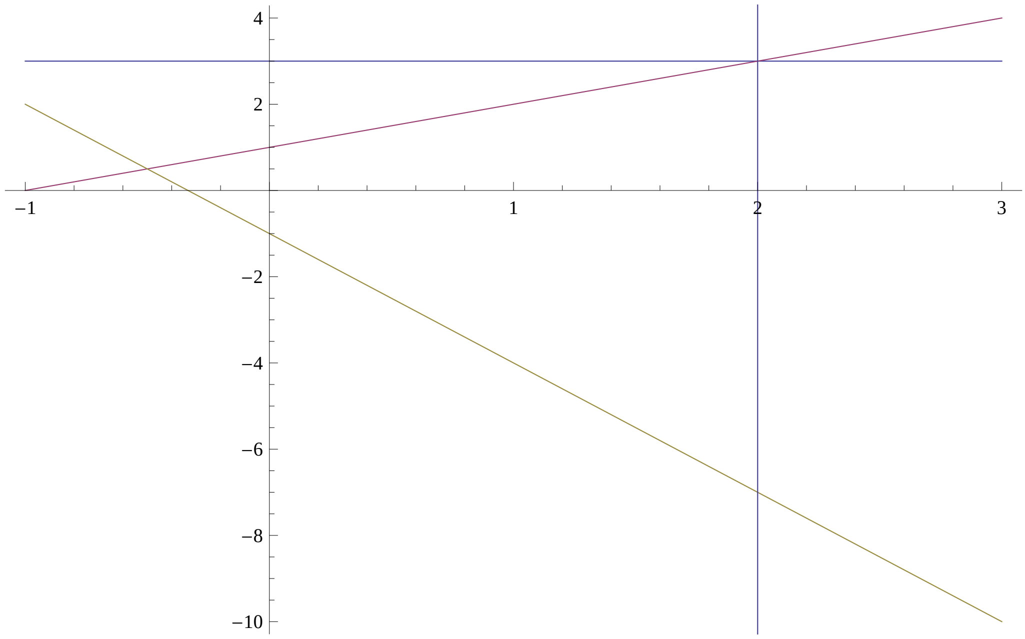

Figure 1: Carefully label each of the four lines here with A., B., C., or D.

[HINT: $y = mx + c$.]

(b) Find the roots of the functions:

(i) $f(x) = 3x^2 - 10x - 8$.

(ii) $g(x) = 3x^2 - 8x + 10$.

(iii) $h(y) = 9y^2 - 6y$.

(iv) $j(x) = x^2 - 10$.

[HINT:

$$x_\pm \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\Bigg]$$

Solution

(b)