MATH6014: Test 1 A (08/03/11)

Name:

Answer all questions. Marks may be lost if necessary work is not clearly shown.

Question 1

(a) Solve for x:

$$\frac{9^{x+2}3^{-2x}}{(\sqrt{3})^x} = \frac{1}{27}.$$

(b) Let

$$f(x) = \log_e e^{42} + 2\log_e 5x + \frac{1}{2}\log_e(9x^4) - 2\log_e(5\sqrt{3}x^2).$$

This function is defined for all $x > 0$. By invoking the log rules and simplifying, show that $f(x)$ is an integer for all $x > 0$ (i.e. $f(x) \in \mathbb{Z}$.).

Solution

(a)

(b)

Question 2

(a) Suppose that a and b are real numbers such that $a + b = 36$ and $a \neq b$. Simplify

$$\frac{a^2 - b^2}{5(a-b) + 6(a-b) + 7(a-b)}.$$

Now, in addition to $a + b = 36$ and $a \neq b$, let $c, d \in \mathbb{R}$ such that $ad - bc \neq 0$. By using the distributive law twice, factorise.

$$a^2d - abc + abd - b^2c$$

Hence, use the no zero divisors theorem to show that $a^2d - abc + abd - b^2c$ is non-zero.

(b) Solve for x:

$$y^3 = \sqrt{\frac{1+x}{1-x}}.$$

Solution

(a)

(b)

Question 3

(a) Consider the following lines defined on the plane:

A. $y = x + 1$.

B. $x = 2$.

C. $y = 1$.

D. $y = -x - 1$.

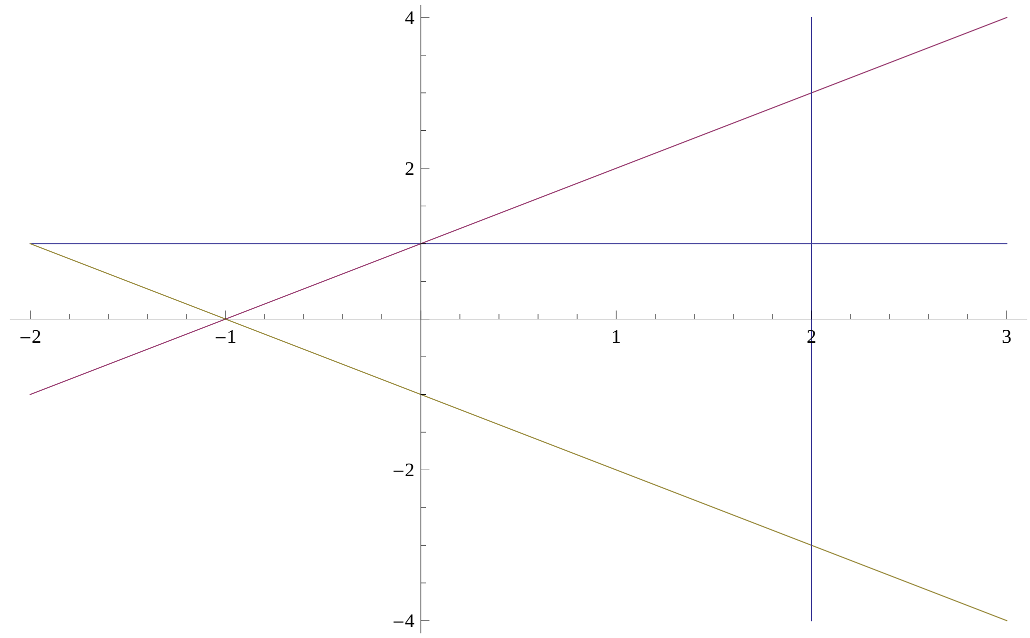

Figure 1: Carefully label each of the four lines here with A., B., C., or D.

[HINT: $y = mx + c$.]

(b) Find the roots of the functions:

(i) $f(x) = 2x^2 - 13x - 7$.

(ii) $g(x) = 2x^2 - 7x + 13$.

(iii) $h(x) = 8x^2 - 16x$.

(iv) $j(y) = y^2 - 5$.

[HINT:

$$x_\pm \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\Bigg]$$

Solution

(b)