MS 2001: Test 2 Sample (08/12/10)

Name:
Student Number:

Answer all questions. Marks may be lost if necessary work is not clearly shown.

Question 1

(a) Where are the following functions differentiable on $\mathbb{R}$? Please quote theorems/ rules used:

(i)
$$f_1(x) = \frac{x^3 + 3x}{x^4 - 1}$$

(ii)
$$f_2(x) = \log x + x^2$$

(iii)
$$f_3(x) = (\sqrt{x^2 + x + 1} + \sin x)^{50}$$

(iv)
$$f_4(x) = |2x - 1|$$

(b) Consider the curve
$$3x^2 + 2x^2y^3 - 9y^2 + y = -21 \tag{1}$$
Show that the point $(1, 3)$ is on the curve. Find the equation of the tangent to the curve at the point $(1, 3)$.

Solution

(a)

(b)

Question 2

Let $g : \mathbb{R} \to \mathbb{R}$ be defined by

$$g(x) = \begin{cases} (x-1)^2 + 1 & \text{if } x < 1 \\ (x-1)^2 + 1 & \text{if } x \geq 1 \end{cases} \tag{2}$$

Show that g is differentiable but not twice differentiable.

Solution

Question 3

1. Suppose that $f : \mathbb{R} \to \mathbb{R}$ is a *twice differentiable* function. Which of the following statements are true? (Circle the correct statement)

   (a) $[f(x)]^2$ is differentiable.

   (b) f cannot be differentiated without the Chain Rule.

   (c) f is differentiable with derivative $f'(x)$ and

   $$\lim_{h \to 0} \frac{f'(a+h) - f'(a)}{h}$$

   exists for all $a \in \mathbb{R}$.

   (d) If f' is the first derivative of f, and f'' is the second, then $f'' = 2f'$.

2. Suppose that $f : [a, b] \to \mathbb{R}$ is a continuous function and has an absolute maximum at $c \in [a, b]$. Which of the following are true? (Circle the correct statement)

   (a) For all $x \in [a, b]$, $f(x) < f(c)$.

   (b) c is a critical point of f.

   (c) For all $x \in [a, b]$, $f(x) \leq c$.

   (d) For all $x \in [a, b]$, $x \leq f(c)$.

3. Which of the following functions satisfy the hypothesis of *Intermediate Value Theorem* on the closed interval $[a, b]$? (Circle the correct statement)

(a) f is differentiable on (a, b) and $f(a) = f(b)$.

(b) f is continuous on (a, b).

(c) f is continuous at a and at b and $f(a) = f(b)$.

(d) f is continuous on $[a, b]$.