MS 2001: Test 1 Sample (27/10/10)

Name:
Student Number:

Answer all questions. Marks may be lost if necessary work is not clearly shown.

Question 1

(a) Find the solution set of the following inequality:

$$\left|\frac{x+1}{5-x}\right| \leq 1$$

(b) Evaluate the following using the Calculus of Limits. You may need the Factor Theorem:

$$\lim_{h\to 1}\frac{h^3-3h+2}{\sqrt{h}-h}$$

Solution

(a)

(b)

Question 2

For a, $b \in \mathbb{R}$ consider the function

$$f(x) := \begin{cases} 4 & \text{for } x \leq 1 \\ ax + b & \text{for } 1 < x < 2 \\ 7 & \text{for } x \geq 2 \end{cases}$$

Determine the values of a and b so that the function f is continuous on $\mathbb{R}$.

Solution

Question 3

1. Suppose that $f : \mathbb{R} \to \mathbb{R}$ is an *even* function. Which of the following statements are true? (Circle the correct statement)

   (a) For all $x \in \mathbb{R}$, $f(-x) = f(x)$.

   (b) For all positive $x \in \mathbb{R}$, there exists a negative $y \in \mathbb{R}$ such that $f(-x) = -f(y)$.

   (c) For all $x \in \mathbb{R}$, $f(-x) = -f(x)$.

   (d) For all $n \in \mathbb{N}$, $f(n)$ is an even number (divisible by 2).

2. Suppose that $f : \mathbb{R} \to \mathbb{R}$ is a *polynomial* function. Which of the following are true? (Circle the correct statement)

   (a) For some a, b, $c \in \mathbb{R}$, $f(x) = ax^2 + bx + c$.

   (b) If $k \in \mathbb{R}$ is a root of $f(x)$ then $f(x - k) = 0$.

   (c) For some $n \in \mathbb{N}$, and $a_n, a_{n-1}, \ldots, a_2, a_1, a_0 \in \mathbb{R}$, with $a_n \neq 0$,

   $$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

   (d) The roots of f are all real numbers.

3. Which of the following statements are true about the absolute value function: $|\cdot| : \mathbb{R} \to \mathbb{R}$? (Circle the correct statement)

(a) For all $x \in \mathbb{R}$, $|-x| = x$.

(b) For all numbers $x, y \in \mathbb{R}$, the triangle inequality, given by

$$|x + y| \geq |x| + |y|,$$

holds.

(c) For all $x \in [-1, 1]$, $|x^2| \leq x$.

(d) For all $x \in \mathbb{R}$, $|x| = |-x|$.